Scale Problems

S1. You have a balance scale and nine coins. One coin is counterfeit and weighs slightly less than the other eight. What is the minimum number of weighings required to identify the counterfeit coin?

S2. You have a balance scale and twelve coins. One coin is counterfeit, but you do not know whether it is too heavy or too light. The other eleven coins weigh the same. What is the minimum number of weighings required to identify the counterfeit coin and determine whether it is too heavy or too light?

S3. You have a balance scale and six balls, of which two are red, two are white, and two are blue. You are told that one ball of each color weighs 10 ounces while the other weighs 11 ounces. What is the minimum number of weighings required to determine which three balls weigh 11 ounces?

S4. You have a balance scale and a set of weights for powers of 3 ounces; that is, one $3^0 = 1$ oz. weight, one 3 oz. weight, one $3^2 = 9$ oz. weight, one $3^3 = 27$ oz. weight, etc. Describe how you can determine the weight of any object whose weight in ounces is a positive integer.

S5. You have a spring scale and ten bags of coins. Each coin is supposed to weigh 10 ounces, but one bag contains counterfeit coins that weigh 11 ounces each. How can you detect which bag contains the counterfeit coins with just one weighing of coins on the spring scale?

S6. Same setting as the previous problem, but now there are an unknown number of bags with counterfeit coins. How can you detect which bags contain the counterfeit coins with just one weighing of coins on the spring scale?

S7. Same setting as the previous problem, but now there are two bags with counterfeit coins. How can you detect which bags contain the counterfeit coins with just one weighing of coins on the spring scale? (The method from the previous problem will work, but there is a more economical solution that requires fewer coins.)
Hat Problems

H1. Ann, Bob, and Cal are lined up in single file. Each is given a hat from a collection of five hats, of which two are red and three are blue. Cal is in the back of the line and can see Ann’s hat and Bob’s hat, but not his own. Bob can see only Ann’s hat, and Ann cannot see any hats. They make the following statements in succession:
   Cal: I don’t know what color my hat is.
   Bob: I don’t know what color my hat is.
   Ann: I know what color my hat is.
What color is Ann’s hat?

H2. Ann, Bob, and Cal each have either a red hat or a blue hat on their head. The hats were placed randomly (one's hat color has no effect on the others) and no person knows the color of his or her hat, but each can see the other two. Once the hats are placed, no communication of any sort is allowed and once they have all seen the others’ hats, they must simultaneously guess the color of their own hat or pass. The three will share a large monetary prize if at least one of them guesses correctly and none guess incorrectly. For example, they could decide that Ann will always say RED and the others will always PASS. This will yield the money half the time. Devise an optimal strategy.

H3. Ann, Bob, and Cal are each given a hat with a positive integer on it and are told that one of the integers is the sum of the other two. Each person can see only the numbers on the others’ hats. They make the following statements in succession:
   Ann: I cannot deduce what my number is.
   Bob: Knowing that, I still cannot deduce what my number is.
   Cal: Knowing that, I still cannot deduce what my number is.
   Ann: Now I can deduce that my number is 50.
Assuming that they all use sound reasoning, what are the numbers on Bob’s and Cal’s hats?

H4. Same setting as the previous problem, but Ann deduces that her number is 60. Find all possible solutions.