1. (E) If $A = \frac{\pi}{3}$, for example, then (e) is false, since $\frac{1}{4} \neq \frac{3}{4}$; the rest are identities.

2. (B) There are $(10)(4)(6) = 240$ choices.

3. (A) 4% of 300% of x is equal $(.04)(3.00x) = .12x$

4. (B) A 4-digit number divisible by 5 can have:
9 possible thousands digits, 10 possible hundreds digits, 10 possible tens digits, and 2 possible units digits. Hence, there are $(9)(10)(10)(2) = 1800$ 4-digit numbers divisible by 5.

5. (C) # of ways to choose 3 out of 30 $= {}_{30}C_3 = \frac{30!}{27!3!} = \frac{30\times29\times28}{6} = (10)(29)(14) = 4060$. The chances of winning are 1 out of 4060.

6. (D) From the information given, $c^2 = a^2 + b^2$, $c > a$, and $c > b$. Thus,
$$c^3 = c \cdot c^2 = c \cdot a^2 + c \cdot b^2 > a \cdot a^2 + b \cdot b^2 = a^3 + b^3,$$
so $a^3 + b^3 < c^3$, (d) is correct, and (a) and (c) are false. To see that (b) and (e) are also false, note that $a^3 - b^3 < a^3 + b^3 < c^3$.

7. (A) Both men travel the same distance, $2x$. Both men row at the same rate, $r$ and the river which Bill is rowing on has a current of $c (< r)$. Since distance = rate × time, we have time = distance/rate.
Bill's time $= \frac{x}{r+c} + \frac{x}{r-c} = \frac{xr-xc+xr+xc}{r^2-c^2} = \frac{2xr}{r^2-c^2}$ and Bob's time $= \frac{2x}{r}$
Since $r^2 > r^2 - c^2$ implies $\frac{r}{r^2-c^2} > \frac{1}{r}$, we have $\frac{2xr}{r^2-c^2} > \frac{2x}{r}$ and Bill's time is greater than Bob's time.

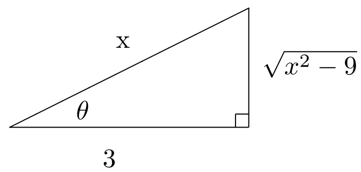
8. (D) There are $(3)(2^9) = (3)(512) = 1536$ ways.

9. (B) $12 + 8 + \frac{16}{3} + \frac{32}{9} + \cdots = a + ar + ar^2 + ar^3 + \cdots$, a geometric series with a = 12 and $r = \frac{2}{3}$ giving an infinite sum of $\frac{12}{1-\frac{2}{3}} = \frac{12}{\frac{1}{3}} = 36$.

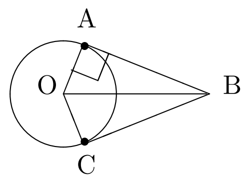
10. (E) If $p - q > 0$, then $p > q$ and none of (a),(b), (c), or (d) are necessarily true.

11. (E) If $y$ varies directly with a function of $x$, then $y = kf(x)$ for some constant $k$ and if $y$ varies inversely with a function of $x$, then $y = \frac{c}{f(x)}$ for some constant $c$.
$y = \frac{10^{\log x}}{x^2} = \frac{x}{x^2} = \frac{1}{x}$, since $10^{\log x} = 10^{\log_{10} x} = x$ and $y$ varies inversely with $x$.

12. (C) Suppose that $0 < \theta < \frac{\pi}{2}$ and $\sec\theta = \frac{x}{3} = \frac{hyp}{adj}$.
From the corresponding right triangle, $\csc\theta = \frac{hyp}{opp} = \frac{x}{\sqrt{x^2-9}}$.




13. (C) Consider the drawing of the situation below. The tangents to the circle form right angles with the radii drawn to the points A and C. This yields two right triangles each with one leg = 1 and hypotenuse = 3. Then, the second leg is $\sqrt{3^2 - 1^2} = \sqrt{9-1} = \sqrt{8}$.
The area of the quadrilateral OABC = 2(area of one of the triangles) $= 2(\frac{1}{2}(b)(h)) = (b)(h) = (1)(\sqrt{8}) = 2\sqrt{2}$.



14. (B) Consider the statement "Not all dogs have fleas." applied to 3 possible sets of dogs. A = {every dog has fleas}, B = {no dog has a flea}, and C = {some dogs do and some do not have fleas}. The truth value of "Not all dogs have fleas" is False, True, True, respectively. Only the statement, "Some dogs do not have fleas" has the same truth values when applied to the 3 possible sets.

15. (A) P(at least 1 caramel) = P(1st piece is caramel and 2nd piece is not) + P(1st piece is not caramel and 2nd piece is) + P(both pieces are caramel) = $(\frac{4}{10})(\frac{6}{9}) + (\frac{6}{10})(\frac{4}{9}) + (\frac{4}{10})(\frac{3}{9}) = \frac{60}{90} = \frac{2}{3}$.

(Alternatively, P(at least 1 caramel) = 1 − P(no caramels) = $1 - (\frac{6}{10})(\frac{5}{9}) = 1 - \frac{30}{90} = 1 - \frac{1}{3} = \frac{2}{3}$.)

16. (E) There are ${}_9C_6 = \frac{9!}{6!3!} = \frac{9\times8\times7}{3\times2} = 84$ ways to pick six workers out of the nine. The number of teams with both Tom and Joe is ${}_7C_4 = \frac{7!}{4!3!} = \frac{7\times6\times5}{3\times2} = 35$. Therefore, the number of teams that Tom and Joe are not both on is 84 - 35 = 49.

17. (C) Since the values for x are the nonnegative integers, the function has the values:
$\lambda$, $\frac{\lambda}{e^{\lambda}}$, $\frac{\lambda}{2e^{2\lambda}}$, $\frac{\lambda}{3!e^{3\lambda}}$, ... and since $e \approx 2.7$ and $\lambda > 0$, $e^{\lambda} > 1$, making $f(x)$ a strictly decreasing function. Therefore, the maximum value occurs at x = 0.

18. (B) We can think of our functions as permuting the vertices in the following way:
$s : (A, B, C) \mapsto (A, C, B)$ $\quad$ $r : (A, B, C) \mapsto (B, C, A)$ $\quad$ $t : (A, B, C) \mapsto (C, A, B)$

Then the correct answer is $(b)$ since
$(s \circ r \circ s \circ r)(A, B, C) = s(r(s(B, C, A))) = s(r(B, A, C)) = s(A, C, B) = (A, B, C)$.

Note that none of the other functions bring us back to the original position of our triangle:
$(s \circ t)(A, B, C) = s(C, A, B) = (C, B, A)$
$(t \circ s)(A, B, C) = t(A, C, B) = (B, A, C)$
$(s \circ r \circ s \circ t)(A, B, C) = s(r(s(C, A, B))) = s(r(C, B, A)) = s(B, A, C) = (B, C, A)$
$(r \circ r)(A, B, C) = r(B, C, A) = (C, A, B)$

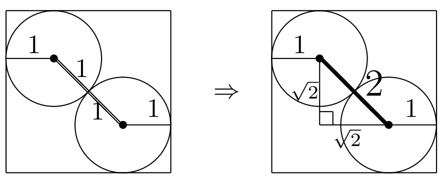
19. (C) Since $x$ is an odd natural number, when divided by six, the only possible remainders are $y = 1$, 3, or 5. Therefore $y^2 = 1, 9$, or 25. When each of those values for $y^2$ is divided by 4, the remainder, $z$, equals 1.

20. (D) Note that the 3 diagonals that connect opposite vertices together all meet in exactly one point. They also divide our hexagon into 6 regions. Each of the 6 regions is divided into 4 non-overlapping regions by the remaining diagonals, leaving a total of 24 regions.

21. (C) $15! = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 13 \cdot 11 \cdot 7^2 \cdot 5^3 \cdot 3^6 \cdot 2^{11}$.

Therefore, the largest perfect square is $7^2 \cdot 5^2 \cdot 3^6 \cdot 2^{10}$.

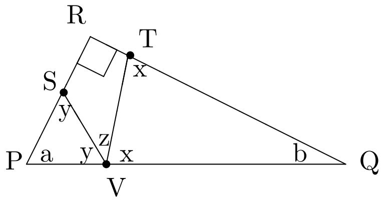
22. (E) From the diagram, the side of the square has length $= 1 + \sqrt{2} + 1 = 2 + \sqrt{2}$.
Therefore, the area $= (2 + \sqrt{2})^2 = 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2}$.




23. (B) Given two solutions $a$, $b$ we have $0 = (x - a)(x - b) = x^2 - (a + b)x + ab$. Since the sum of the two solutions is $\frac{11}{12}$ and the product of the two solutions is $\frac{1}{6}$, one possible quadratic equation would be: $x^2 - \frac{11}{12}x + \frac{1}{6} = 0$
Multiplying by 12 yields another possible equation: $12x^2 - 11x + 2 = 0$.

24. (E) Primes less than 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
Numbers under 30 which are a product of two distinct primes:
$2 \times 3 = 6, 2 \times 5 = 10, 2 \times 7 = 14, 2 \times 11 = 22, 2 \times 13 = 26,$
$3 \times 5 = 15, 3 \times 7 = 21.$
There are 7 such numbers.

25. (D) With $\overline{QT} \cong \overline{QV}$, and $\overline{PS} \cong \overline{PV}$, we can label the triangle with angles $x, y, z, a$, and $b$ as




This lead to the following system of equations:

$a + b + 90 = 180$

$a + 2y + b + 2x = 360$

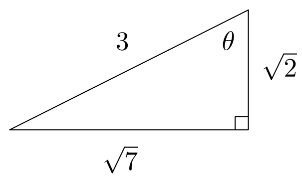
$x + y + z = 180$

Solving the first equations yields $a + b = 90$. Substitute into the second equation to get $90 + 2y + 2x = 360 \implies 2y + 2x = 270 \implies x + y = 135$. Finally, substituting into the last equation yields $135 + z = 180 \implies z = 45$.

26. (B) Since the two roots are $r_1$ and $r_2$, we have $(x - r_1)(x - r_2) = x^2 - (r_1 + r_2) + r_1 r_2 = x^2 - px + q$. So, $p = r_1 + r_2$ and $q = r_1 r_2$.
$p^2 = (r_1 + r_2)^2 = r_1^2 + 2r_1 r_2 + r_2^2$, then $r_1^2 + r_2^2 = p^2 - 2r_1 r_2 = p^2 - 2q$.

27. (C) Since $N - 1$, $N$, and $N + 1$ are consecutive positive integers, one of them must be divisible by 3, and hence their product is divisible by 3. If $N = 2$ then the product is 6 and the other answers are false.

28. (A) $\cos(2 \sin^{-1} \frac{\sqrt{7}}{3}) = \cos^2 \theta - \sin^2 \theta$ where $\theta = \sin^{-1} \frac{\sqrt{7}}{3}$. From the right triangle below, we have $\cos \theta = \frac{\sqrt{2}}{3}$ and $\sin \theta = \frac{\sqrt{7}}{3}$. This gives
$\cos(2 \sin^{-1} \frac{\sqrt{7}}{3}) = \cos^2 \theta - \sin^2 \theta = \frac{2}{9} - \frac{7}{9} = -\frac{5}{9}$




29. (C) A fair coin tossed until a head appears on an odd-numbered tossed can happen in the following ways:

H, TTH, TTTTH, TTTTTTH, ...

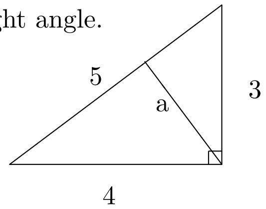
The probability of getting one of those scenarios is $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + ...$ This is a geometric series with $a = \frac{1}{2}$ and $r = \frac{1}{4}$ whose sum $= \frac{1/2}{1-1/4} = \frac{1/2}{3/4} = \frac{2}{3}$.

30. (D) The number of ways to arrange 8 letters = 8! but, since some letters are indistinguishable (two N's and three A's), not all arrangements are distinct. For any given arrangement, there are 2! = 2 ways to arrange the two N's and 3! = 6 ways to arrange the three A's, resulting in $2 \times 6 = 12$ indistinguishable arrangements.
Overall, the total number of distinct arrangement must be $\frac{8!}{2! \times 3!} = 8 \times 7 \times 5 \times 4 \times 3 = 3360$

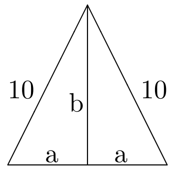
31. (A) P(able to cross) = P(only 4 rocks are needed) + P(all 5 rocks are needed)
P(only 4 rocks are needed) = P(4 successes in a row)
P(all 5 rocks are needed) = P(1 failure and 4 successes)$\times$ 4
Multiply by 4 to represent the possible locations for the single failure.
P(only 4 rocks are needed) + P(all 5 rocks are needed) = $(\frac{2}{3})^4 + 4(\frac{2}{3})^4(\frac{1}{3}) = \frac{16}{81} + \frac{64}{243} = \frac{48}{243} + \frac{64}{243} = \frac{112}{243}$.

32. (E) 100! has **20** multiples of 5 and **4** multiples of 25. Hence, the exponent on 5 in the prime factorization is 20+4 = 24 and the exponent on 2 is > 50 since there are 50 even numbers. Each 2×5 product results in a zero and there will be 24 such products, so 100! will end in 24 zeros.

33. (A) Let $a$ equal the length of the altitude from the vertex at the right angle. From the picture, Area $= \frac{1}{2}(4)(3)$ but area also equals $\frac{1}{2}(5)(a)$. So, $12 = 5a$ and $a = \frac{12}{5} = 2.4$ is the length.




34. (D) Draw an isosceles triangle with height $b$ and base $2a$. The third side(the base) must be a whole number strictly between 0 and 20 making $a = \frac{1}{2}, 1, \frac{3}{2}, \ldots, 9$, or $\frac{19}{2}$. However, there are two further restriction involving $a$.



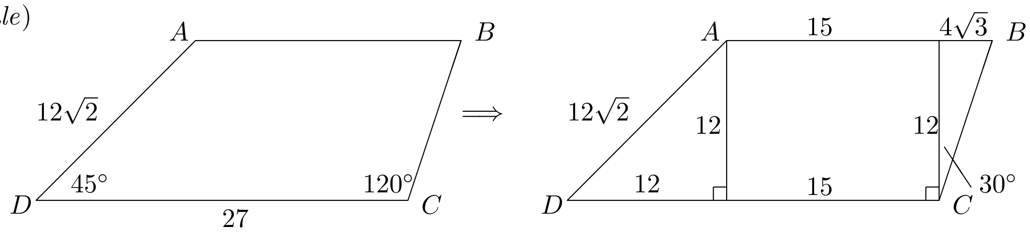

$a^2 + b^2 = 100$ and Area $= \frac{1}{2}(2a)(b) = ab$ must be a whole number. But with the first equation, $b = \sqrt{100 - a^2}$, and therefore the area $= a\sqrt{100 - a^2}$. Only two assignments of $a$ will satisfy the "area is a whole number" condition: $a = 6$ or $a = 8$. Regardless, the area is $(6)(8) = 48$ square inches.

35. (B) $\log_3(x+1) - \log_9 x = 1$. Then, $9^{\log_3(x+1) - \log_9 x} = 9^1 \Longrightarrow \frac{9^{\log_3(x+1)}}{9^{\log_9 x}} = 9 \Longrightarrow \frac{(3^2)^{\log_3(x+1)}}{x} = 9 \Longrightarrow$ $3^{2\log_3(x+1)} = 9x \Longrightarrow 3^{\log_3(x+1)^2} = 9x \Longrightarrow (x+1)^2 = 9x \Longrightarrow x^2 + 2x + 1 = 9x \Longrightarrow x^2 - 7x + 1 = 0 \Longrightarrow$ $x = \frac{7 \pm \sqrt{49-4}}{2} \Longrightarrow x = \frac{7 \pm \sqrt{45}}{2}$. Both values are positive so both satisfy the original equation.

36. (B) Let $x =$ age of the $11^{th}$ person, The sum of the ages for the first 10 people is $10(20.5) = 205$. The sum of the ages for the 11 people is given by $205 + x$ and the average is $\frac{205+x}{11} = 21$. Multiply by 11 to get $205 + x = 231 \Longrightarrow x = 26$

37. (D) $\sin\theta + \cos\theta = \frac{\sqrt{3}}{\sqrt{2}} \Longrightarrow (\sin\theta + \cos\theta)^2 = (\frac{\sqrt{3}}{\sqrt{2}})^2 \Longrightarrow \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{3}{2} \Longrightarrow$ $1 + 2\sin\theta\cos\theta = \frac{3}{2} \Longrightarrow 2\sin\theta\cos\theta = \frac{1}{2} \Longrightarrow \sin 2\theta = \frac{1}{2} \Longrightarrow 2\theta = 30°, 150°, 390°, ... \Longrightarrow \theta = 15°, 75°, 195°, ...$ The smallest positive angle is 15°.

38. (A) Quadrilateral $ABCD$ with $\overline{CD} \parallel \overline{AB}$, measure of angle D $= 45°$, measure of angle C $= 120°$, $AD = 12\sqrt{2}$ and $CD = 27$ looks like
(*not drawn to scale*)




Therefore, $AB = 15 + 4\sqrt{3}$ and Area $= \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(12)(27 + 15 + 4\sqrt{3}) = 6(42 + 4\sqrt{3}) = 252 + 24\sqrt{3}$.

39. (B) There are $6! = 720$ ways to position the 6 numbers. 24 of these arrangements result in a magic triangle, as we show below, for a probability of $\frac{24}{720} = \frac{1}{30}$. Since some two numbers must be on the same edge as 6, the smallest possible edge sum for a magic triangle is $1 + 2 + 6 = 9$, as in the given triangle. Similarly, since some two numbers must be on the same edge as 1, the largest possible edge sum for a magic triangle is $1+5+6 = 12$. Thus, if $E$ represents the edge sum for a magic triangle, then $9 \leq E \leq 12$. Notice that if we add the three edge sums in a magic triangle, each vertex number is added twice, so that $3E = 21 + V$, where $V$ is the sum of the vertex numbers. Thus, if $E = 9$, then $V = 6$ and the three vertex numbers must be 1, 2, and 3, as in the given triangle. There are a total of $3! = 6$ magic triangles with $E = 9$, obtained by rotating or reflecting the given triangle. If $E = 10$ then $V = 9$ and the vertex numbers must be 1, 3, and 5 in order to get a magic triangle. Once again there are 6 arrangements of these vertex numbers; each arrangement determines a unique magic triangle with $E = 10$. If $E = 11$ then $V = 12$ and the vertex numbers must be 2, 4, and 6 in order to get a magic triangle; this leads to 6 more magic triangles. Finally, if $E = 12$ then $V = 15$ and the vertex numbers must be 4, 5, and 6; this yields 6 more magic triangles, for a total of 24.

40. (C) When Greg first leaves his home, he starts from a stopped position and gradually increases his speed down the hill. So, the distance from his house starts out increasing very slowly. Upon coming to a stop at the traffic light, he must reduce his speed slowly, so that his distance is again increasing very slowly. ($c$) is the only graph that represents this scenario.