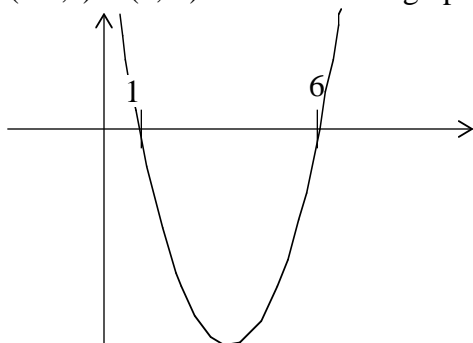


**CIPHERING PROBLEMS SOLUTIONS**  
**HIGH SCHOOL MATHEMATICS TOURNAMENT**  
**ARMSTRONG ATLANTIC STATE UNIVERSITY-FEBRUARY 14, 1998**

1. ANSWER: interval notation:  $(-\infty, 1) \cup (6, \infty)$ ; set notation:  $\{x : x < 1 \text{ or } x > 6\}$

The given inequality is equivalent to  $x^2 - 7x + 6 > 0$ . Since  $x^2 - 7x + 6 = (x - 6)(x - 1)$  we can see that the solution is  $(-\infty, 1) \cup (6, \infty)$  either from the graph of  $y = x^2 - 7x + 6$  or from a sign chart:



$x - 6$	-		-	+
$x - 1$	-		+	+
$x^2 - 7x + 6$	+		-	+
		1		6

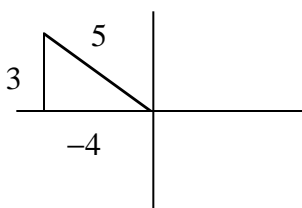
2. ANSWER: 2

Using a property of logarithms, we get  $\log_2 x(x + 6) = 4$ . Rewriting in exponential form, the equation becomes  $x(x + 6) = 2^4 \Rightarrow x^2 + 6x = 16 \Rightarrow x^2 + 6x - 16 = 0$ . The two solutions to this equation are  $-8$  and  $2$ . The solution  $-8$  is extraneous.

3. ANSWER:  $-\frac{24}{25}$

By the double angle formula for the sine function,  $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ . From the reference triangle for  $\alpha$  given below and using the Pythagorean Theorem to complete that triangle, we see that  $\cos \alpha = -\frac{4}{5}$ . We could have used the identity  $\sin^2 \alpha + \cos^2 \alpha = 1$  and the fact that  $\cos \alpha < 0$  to find  $\cos \alpha$ . Therefore,

$$\sin 2\alpha = 2 \left( \frac{3}{5} \right) \left( -\frac{4}{5} \right) = -\frac{24}{25}.$$



4. ANSWER:  $2\sqrt{5}$  feet

Let  $x$  be the length, in feet, of one of the sides. Then the length of the other side is  $x + 1$  feet. Since the area is one square foot we have the equation  $x(x + 1) = 1 \Rightarrow x^2 + x - 1 = 0$ . Using the quadratic formula we get  $x = \frac{-1 \pm \sqrt{5}}{2}$ . The solution  $x = \frac{-1 - \sqrt{5}}{2}$  is negative, and, therefore, does not make sense as a length.

Therefore, one of the sides has length  $\frac{-1 + \sqrt{5}}{2}$  feet and the other has length  $\frac{-1 + \sqrt{5}}{2} + 1$  feet =  $\frac{1 + \sqrt{5}}{2}$  feet.

Hence, the perimeter has a length, in feet, of  $2 \left( \frac{-1 + \sqrt{5}}{2} \right) + 2 \left( \frac{1 + \sqrt{5}}{2} \right) = -1 + \sqrt{5} + 1 + \sqrt{5} = 2\sqrt{5}$

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5. ANSWER: 125%

The volume  $V$  of a right circular cylinder is given by  $V = \pi r^2 h$ , where  $r$  is the radius and  $h$  is the height of the cylinder. If the radius  $r$  is increased by 50%, then the new radius  $r'$  is  $1.5r$ . Therefore, the new volume  $V'$  is given by the following:  $V' = \pi(r')^2 h = \pi(1.5r)^2 h = 2.25\pi r^2 h = 2.25V$ . Hence,  $V'$  is 225% of  $V$ ; an increase of 125%.

6. ANSWER:  $-\frac{3}{2}$

$$f(0) = 1 \Rightarrow c = 1$$

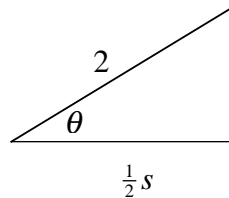
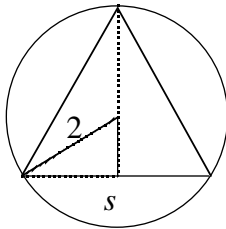
$$f(1) = 3 \Rightarrow a + b + 1 = 3 \Rightarrow a + b = 2$$

$$f(2) = 2 \Rightarrow 4a + 2b + 1 = 2 \Rightarrow 4a + 2b = 1$$

We get the solution by solving the system  $\begin{cases} a + b = 2 \\ 4a + 2b = 1 \end{cases}$  simultaneously. This can be done, for example, by multiplying both sides of the first equation by  $-2$  and adding this new equation to the second equation. The result is  $2a = -3 \Rightarrow a = -\frac{3}{2}$ .

7. ANSWER:  $2\sqrt{3}$  inches

The shaded triangle in the first figure is enlarged in the second figure. Since the interior angle  $\theta$  is  $30^\circ$  and  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ , we have that  $\frac{\frac{1}{2}s}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{1}{2}s = \sqrt{3} \Rightarrow s = 2\sqrt{3}$ .



8. ANSWER: 25

Let  $n$  be the number of students taking the test. Then  $75 = \frac{\text{original sum of scores}}{n}$  and

$$75.4 = \frac{\text{new sum of scores}}{n} = \frac{\text{original sum of scores} + 10}{n}. \text{ Let } s \text{ be the original sum of scores. Since } s = 75n,$$

$$\text{we have that } 75.4 = \frac{75n + 10}{n} \Rightarrow 75.4n = 75n + 10 \Rightarrow .4n = 10 \Rightarrow n = \frac{10}{.4} = 25$$

9. ANSWER:  $\frac{27}{100}$

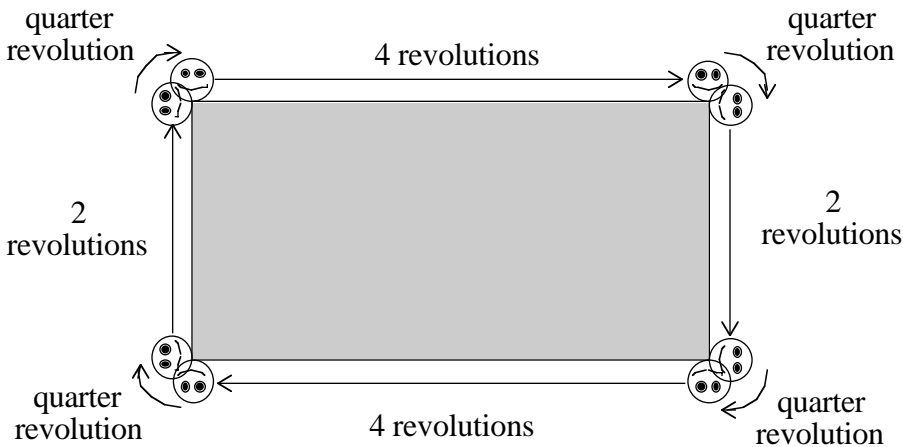
We assume that the numbers are chosen one at a time. Then, either (a) the first two numbers are the same, (b) the first and last numbers are the same, or (c) the last two numbers are the same. In case (a), there are 10 choices for the first selection, 1 choice for the second (it must be the same as the first) and 9 choices for the last selection (it must be different from the number already chosen). Hence, case (a) has  $10 \cdot 1 \cdot 9 = 90$  possible ways of happening. Similar arguments show that in case (b) there are  $10 \cdot 9 \cdot 1 = 90$  possibilities, and case (c)

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also has  $10 \cdot 9 \cdot 1 = 90$  possibilities. Hence, there are 270 possible ways to choose the numbers in the desired fashion. There are  $10 \cdot 10 \cdot 10 = 1000$  total ways to choose the three numbers without restrictions. Hence, the probability is  $\frac{270}{1000}$ .

10. ANSWER: 13

The following diagram describes the movement of the coin along its journey.



11. ANSWER: 5

Since  $-1$  is a triple root,  $(x+1)^3$  must be a factor of  $x^4 + ax^3 + bx^2 + cx + 2$ . The remaining factor must be  $x+2$ . Expanding  $(x+1)^3$  using the binomial theorem results in the polynomial  $x^3 + 3x^2 + 3x + 1$ . Therefore,  $x^4 + ax^3 + bx^2 + cx + 2 = (x^3 + 3x^2 + 3x + 1)(x+2) = x^4 + x^3 \cdot 2 + 3x^2 \cdot x + \dots$ . Hence, the  $x^3$ -term is  $5x^3$ .

12. ANSWER: 144 miles per hour

The basic equation is distance = rate  $\cdot$  time. The driver's average speed is given by  $\frac{\text{distance}}{\text{time}}$ . Suppose that one lap is  $x$  miles. Then, the times to drive the two laps are  $\frac{x}{180}$  and  $\frac{x}{120}$ , respectively. Therefore, the driver's average speed is  $\frac{2x}{\frac{x}{180} + \frac{x}{120}} = \frac{2x}{\frac{2x}{360} + \frac{3x}{360}} = \frac{360(2x)}{5x} = 72 \cdot 2 = 144$ .