Errata et Cetera

for the first printing of

Differential Equations with Boundary Value Problems

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Most of what's here are errors of substance. Minor typographical glitches are included here only if they have the potential to cause confusion.

Front endpapers, left side, line 4

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

Section 1.2, page 12, line 8

$$\int_{t_0}^{t} \frac{y'(s)}{y(s)}\,ds = \int_{t_0}^{t} k\,ds$$

Section 1.2, page 12, Problem 11

...15 counts per minute per gram of carbon. *(not per second)*

Section 2.1, page 32, line 4

$$y = y_0\,e^{-k\,(t-t_0)} + \int_{t_0}^{t} e^{-k\,(t-s)} f(s)\,ds.$$

Section 2.2.1, page 34 *I had the physics all wrong on the rocket thrust problem. The following is a whole new page 34. (The only real change to Example 1 is the value of k.)*

Linear Drag, Variable Mass, and Thrust If the mass of the projectile varies with time and an additional force f is present, then the differential equation is still linear and first order:

$$(m\,v)' = -m\,g - k\,v + f, \quad \text{or} \quad m\,v' + (k+m')\,v = -m\,g + f.$$

In the case of a fuel-burning rocket, it turns out that $f = (v+\mu)m'$, where μ is the velocity at which burnt fuel is expelled relative to the rocket. The quantity $T = \mu\,m'$ is *thrust*, and the differential equation reduces to

$$m\,v' + k\,v = -m\,g + T.$$

Example 1 A small rocket with mass 100 kg, including 30 kg of fuel, is fired vertically from rest at time $t = 0$. Its engine provides a thrust of 1000 N and burns fuel at a steady rate of 1 kg/s. Find the velocity and height attained by the rocket by the time its fuel is spent. Assume constant gravitational acceleration $g = 9.8$ m/s^2 and a known drag coefficient $k = 2$ N s/m.

Solution: The mass of the rocket satisfies $m'(t) = -1,\ m(0) = 100$; therefore, $m(t) = 100 - t$ for $0 \le t \le 30$ s. Thus the equation of interest is

$$\frac{dv}{dt} + \frac{2}{100-t}\,v = -9.8 + \frac{1000}{100-t}, \quad 0 \le t \le 30.$$

The integrating factor is $(100-t)^{-2}$; therefore

$$\frac{d}{dt}\left(\frac{v}{(100-t)^2}\right) = -\frac{9.8}{(100-t)^2} + \frac{1000}{(100-t)^3}.$$

Integration now yields

$$v = (100 - t)^2 \left(-\frac{9.8}{100 - t} + \frac{500}{(100 - t)^2} + C \right)$$
$$= -9.8\,(100 - t) + 500 + C\,(100 - t)^2.$$

The initial condition $v(0) = 0$ gives $C = 0.048$; therefore,

$$v = 9.8t - 480 + 0.048\,(100 - t)^2 = 0.048\,t^2 + 0.2\,t.$$

Now we integrate and use $y(0) = 0$ to obtain the height

$$y = 0.016\,t^3 + 0.1\,t^2.$$

Thus, after 30 seconds the rocket has reached a height of 522 m and has a velocity of 49.2 m/s. Once the fuel has been spent, the mass will remain constant, and there will be no more thrust; therefore, the velocity of the rocket for the remainder of its flight will obey the simpler equation

$$\frac{dv}{dt} + \frac{2}{70}\,v = -9.8.$$

Section 2.2.1, page 35, Problem 3

Change the value of k *to* $k = 0.09$.

Section 2.3, page 46, middle of the page

Note that if p and f are continuous on I, then ...

Section 3.2, page 60, Example 1

The graph doesn't match the solution here. (Who'd notice?) We'll change the differential equation and keep the graph. The three displayed equations are then as follows:

$$\frac{dy}{dt} = \frac{1 + \cos t}{1 + 2y}$$

$$(1 + 2y)\,dy = (1 + \cos t)\,dt\,.$$

$$y + y^2 = t + \sin t + C\,.$$

Section 3.2, page 63, Problem 9

In part (a) the interval should be half closed: $(-\infty, T]$.

Section 3.2, page 64, Problem 18

18. $y' = -\sin^2(t + y + 1) \quad (z = t + y + 1)$

Section 3.3, page 65, Example 2

The last three displayed equations should be:

$$e^t u - e^{t_0}/y_0 = t - t_0, \qquad u = \frac{e^{t_0} + y_0(t - t_0)}{y_0\,e^t}, \qquad y = \frac{y_0\,e^t}{e^{t_0} + y_0(t - t_0)}$$

Section 3.4, pp. 69-70, Example 3

Change the first two displayed equations in Example 3 to:

$$y'' = \frac{y'}{y}(y^2 - y') \qquad \frac{dv}{dy} + \frac{1}{y}\,v = y$$

Section 3.4, page 71, Problem 15

(b) Show that an object that falls from rest through a distance z in a vacuum achieves a speed increase of $\sqrt{2\,g\,z}$.

Section 3.4, page 72, Problem 18

18. Let $0 < \theta_0 < \pi$, and suppose that the pendulum in Problem 17 is set in motion with

$$\theta(0) = \theta_0 \quad \text{and} \quad \omega(0) = 0.$$

Let T be the period of the subsequent motion of the pendulum.

(a) Show that, for $0 \le t \le T/2$,

$$\omega = -\sqrt{\frac{2\,g}{L}}\,\sqrt{\cos\theta - \cos\theta_0}\,,$$

and therefore

$$-\sqrt{\frac{2\,g}{L}}\,dt = \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}\,.$$

(b) Note that $t = T/4$ is then the time at which the pendulum first passes the downward vertical position (i.e., $\theta(T/4) = 0$). By integrating ...

Section 3.5.1, page 75

Variable Mass and Thrust The incorporation of variable mass m and thrust T into the hybrid drag model gives us the equation

$$m\,\frac{dv}{dt} = -m\,g + T - k_\ell\,v - k_q\,v\,|v|\,,$$

which upon rearrangement becomes

$$v' + \frac{k_\ell}{m}\,v = -g + \frac{T - k_q\,v\,|v|}{m}\,. \tag{6}$$

Section 3.5.1, page 76

Example 1 Consider a small rocket whose velocity is governed by (6) with

$$m = 0.1 + e^{-t}\ \text{kg}, \quad T = 500\,e^{-t}\ \text{N},$$
$$k_\ell = 0.01\ \text{N s/m}, \quad k_q = 10^{-4}\ \text{N s}^2/\text{m}^2;$$

that is,

$$v' + \frac{0.01\,v}{0.1 + e^{-t}} = -9.8 + \frac{500\,e^{-t} - 10^{-4}\,v\,|v|}{0.1 + e^{-t}}\,.$$

Section 3.5.1, page 77, Problem 9

Replace the whole thing with:

9. An object is moving in a viscous fluid and is either accumulating or ejecting mass that moves at a velocity of μ m/s relative to the object. Assuming quadratic drag and that no gravitational force is present, we then have the differential equation

$$m(t)\frac{dv}{dt} = -k_q\, v\,|v| + \mu\, m'(t).$$

Suppose that the object's mass is given by $m = m_0 + r\,t$. (The equation is then separable.) Find a general solution in each of the following cases:

(a) $v(t) \geq 0$ and $r\mu > 0$ (b) $v(t) \geq 0$ and $r\mu < 0$

(c) $v(t) \leq 0$ and $r\mu > 0$ (d) $v(t) \leq 0$ and $r\mu < 0$

Section 4.1, page 88, line -7

Delete the f_y in the last term.

$$y^{(3)}(t_0) = \big(f_{tt} + f_{yt}\, y' + f_y\, y'' + f_{ty}\, y' + f_{yy}\, y'y' + y''\big)\Big|_{t=t_0}$$

Section 4.1, page 89

Example 1 ends on line 9. Line 10 begins a new paragraph.

Section 4.2, page 98, Problem 4

4. $\ldots y' = 3\,t\,y^{1/3},\ y(1) = 1.$

Section 4.4, page 114, Problem 8

(d) Explain why a small change in one of the observations can result in a large change in the calculated M if $P_0P_2 - P_1^2$ is small relative to the numerator in formula.

Section 4.4, page 115, Problem 12

(c) Show that the solution $\tilde{u}$ in part (b) satisfies $0 < \frac{1}{\delta} < \tilde{u}(t) < \frac{1}{\gamma}$ for all t and that if u is any other solution of $u' + k\,u = f$, then

$$\lim_{t\to\infty} (u(t) - \tilde{u}(t)) = 0.$$

Section 4.5, page 121

The first row of Table 4 is missing. It's the same as the first row of Tables 1–3.

Section 5.1, page 136, lines -12 to -3

We remark, finally, that the equations discussed so far represent a quite idealized model. In a more complicated setting, it may be that one or more of the damping force, the restorative force of the spring, and the external force depend (perhaps nonlinearly) upon t, y, and/or y'. Moreover, the mass m need not be constant. Under any of these circumstances, the differential equation of the motion has the general form

$$m\,y'' - \rho - \sigma = F + \mu\, m',$$

where the functions ρ and σ represent the damping and restorative forces, respectively, μ is the velocity of the accumulated/lost mass relative to the object, and F accounts for all external forces other than thrust.

Section 5.1, page 139, Problem 11

11. Suppose that for a damped system as in Figure 1, the spring stiffness is $k = 1$ N/m, the damping coefficient is $r = 2$ N·s/m, and the mass is a bucket of sand with a hole in the bottom. The bucket itself has a mass of 1 kg and initially contains 3 kg of sand. Assume that the sand runs out at a constant rate of 0.01 kg/s and a constant velocity of 0.1 m/s relative to the bucket. Write down the differential equation that governs the damped, unforced motion of the spring up to the time when all the sand has run out.

Section 5.2, page 145, Example 6, 4th line

…for any closed box G in $\mathbb{R}^3$.

Section 5.4, page 159, Problem 4

4. Suppose that u and v are defined, but not necessarily differentiable, on an interval I. Show that if there are points t_1, t_2 in I such that $u(t_1) = v(t_1) \neq 0$ and $u(t_2) \neq v(t_2)$, then u and v are linearly independent on I.

Section 5.4, page 161, Problems 30 and 36

30. … $y'' - \frac{2}{t^2}\, y = 0$ …

36. *Change the letter ω to b. (So as not to be inconsistent with Chapter 6.)*

…let $b = -q + p^2/4$.

(b) $b < 0$ (b) $b = 0$ (c) $b > 0$

Section 5.5, page 162, line 2 – equation (1)

… $y(t_0) = 0, \quad y'(t_0) = 0.$

Section 5.5, page 163, line 5 – equation (5)

…Because of the first two equations in (3), the initial conditions $y(t_0) = y'(t_0) = 0$ will be satisfied if we choose a and b such that $a(t_0) = b(t_0) = 0$. Thus we simply integrate from t_0 to t to obtain …

And every integral on the page should have t_0 as the lower limit: $\displaystyle\int_{t_0}^{t}$.

Section 5.6, page 178, Problem 31

31. $9t^2y'' + 3t\,(2 + 3t^2)\,y' + (3t^2 - 2)\,y = 0$

Section 6.1, page 186, line 10

…Theorem 1 of Section 5.4 …

Section 6.2, page 191, Problem 21

The final equation in part (c) should be

$$(y' - 2\,y)^2 (y' + 3\,y)^3 = 5^5 c_1^3 c_2^2.$$

Section 6.3, page 202, Problem 24, line 8

(b) … $r^2 + (a-1)\,r + b$.

Section 6.6, page 217, lines 11–17

iii) For fixed $p, \omega_0 > 0$, the maximum gain is $G = \sqrt{q}/p$, where $q = \omega_0^2 + p^2$.

iv) For any fixed $p, q > 0$, G is a *bounded* function of ω_0 with $\lim_{\omega_0 \to 0^+} G = 1$. Moreover,

- when $0 < q \le p^2/2$, G is a decreasing function of ω_0;
- when $q > p^2/2$, a maximum gain of $G = \frac{2q}{p\sqrt{4q-p^2}}$ occurs when

$$\omega_0 = \sqrt{q - p^2/2}.$$

Section 6.6, page 218, Problems 3, 6, 12

3. $y'' + 9\,y = 9\,(\cos t + \cos 5\,t)$

6. Find a particular solution of

(a) $y'' + 16\,y = 16 \cos 4\,t$ (b) $y'' + y' + 16\,y = 16 \cos 4\,t$

12. (a) … $$y = \frac{\omega^2}{\omega^2 - \omega_0^2} A_0 \left(\cos \omega_0\, t - \cos \omega\, t\right).$$

Section 7.3, page 238, line 4

$$F(s) = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s} = \frac{1 - (1+2s)e^{-2s}}{s^2}$$

Section 7.3, page 241, Problems 30, 32

30. $\dfrac{1 - e^{-s} + e^{-2s}}{s^2 + 4\pi^2}$ 32. $\dfrac{2 - 3e^{-\pi s} + e^{-2\pi s}}{s^2 + 1}$

Section 7.4, page 246, Problems 11, 12

11. $y'' + 4\pi^2 y = 4\pi^2\big(1 - (t - \lfloor t \rfloor)\big)$ 12. $y'' + \pi^2 y = \pi^3\big(t - \lfloor t \rfloor\big)$

Section 7.6, page 253, line -10

$$g * (c\,y) = c\,(g * y) \quad \text{and} \quad g * (u + v) = g * u + g * v.$$

Section 7.6, page 257

Example 8 ends after the third displayed equation.

Section 7.6, page 258, Problem 20

20. $u * v' - u' * v = u_0 v - v_0 u$

Section 8.5, page 292, Problem 31

Two references to Theorem 8.5 should be to Theorem 4.

Section 8.7, page 301, line -6

$$\|\mathbf{x}-\mathbf{a}\| = \left(\sum_{i=1}^{n}(x_i - a_i)^2\right)^{1/2}.$$

Section 9.1, page 319, Problem 14

14. ... $x(0) = 0.25$ and $y(0) = 2$.

Section 9.2, page 329, line 8

... multiplication by Q^{-1} then produces ...

Section 9.3, page 339, line 15

The eigenvalues at $(6,5)$ *are* $-2.5 \pm 10.7\,i$

Section 9.3, page 343, Problem 19(b), line 9

Delete the suggestion.

Section 9.4, page 347, line -20

... in $\mathcal{M}$—at *any* time t_0—remains in $\mathcal{M}$ for all $t \geq t_0$.

Section 9.4, page 348, line 6

Let $\mathcal{M}$ be a closed, bounded, forward-invariant region for (2).

Section 9.4, page 354, Problem 17

(d) Show that the quantity $\varphi(x,y) = \frac{1}{6}\left(3\,y^2 - 6\,x^2 y - 2\,y^3 + 3\,x^2\right)$ is ...

Section 10.2, page 384, Problem 4

$$S' = k\,S - \frac{\beta\,S\,I}{\eta^2 + I^2}, \quad I' = \frac{\beta\,S\,I}{\eta^2 + I^2} - (\gamma + \delta)\,I.$$

Section 10.2, page 385, Problem 9a

$$x(t) = \frac{\beta}{\gamma}\,S(t/\gamma), \quad y(t) = \frac{\beta}{\gamma}\,I(t/\gamma), \quad z(t) = \frac{\beta}{\gamma}\,V(t/\gamma),$$

Section 10.3, page 387, line 5

$$x(t) = \frac{a\,V}{b\,\rho}\,B(\rho\,t/V), \quad y(t) = \frac{1}{b}\,C(\rho\,t/V), \quad k = \frac{a\,\beta\,V}{\rho}, \quad \text{and} \quad q = \frac{C_0}{b}.$$

Section 10.3, page 390

line -12: Let $(\tilde{v}_\sigma, \tilde{w}_\sigma)$ denote the equilibrium point ...

line -3: ... each with the same initial point $(\tilde{v}_0, \tilde{w}_0)$, which ...

Section 10.4, page 399

lines 8–10: Should be parts (c) and (d).

line 18: Reference to Problem 7 should be to Problem 14.

Section 10.5, page 405

line -10*:* $m_0\,G = 26.67$

line -3*:* $(x_1(0), y_1(0)) = (51, 16.5)$ and $(x_2(0), y_2(0)) = (40, 0)$

line -1*:* $(x_1'(0), y_1'(0)) = (0, 0)$ and $(x_2'(0), y_2'(0)) = (0, 0.5)$.

Section 10.5, page 407, Problem 6

(d) Show that substitution of $r = u^{-1}$ leads to the equation $\frac{d^2u}{d\theta^2} + u = k$, where ...

Section 11.4, page 440

Example 1 ends before the last paragraph.

Section 11.4, page 443

Example 3 continues until Figure 4.

Section 11.5, page 450, line 13

...(See also Problem 8.)

Section 11.5, page 453, line -5

...that of Theorem 2...

Section 11.5, page 455, footnote

...A symmetric matrix A is ...

Section 12.1, page 475, lines 7 and 11

y should be u.

Section 12.1, page 476, line 2

y should be u.

Section 12.1, page 478, line 3

y should be u.

Section 12.1, page 480, line -6

y should be u.

Section 12.3, page 499, lines −3, −5, −7

The k is missing in each occurence of $e^{-k\left(\frac{m^2}{\ell^2}+\frac{n^2}{h^2}\right)\pi^2 t}$ *:*

$$T_{mn}(t) = e^{-k\left(\frac{m^2}{\ell^2}+\frac{n^2}{h^2}\right)\pi^2 t}.$$

$$T_{mn}X_mY_n = e^{-k\left(\frac{m^2}{\ell^2}+\frac{n^2}{h^2}\right)\pi^2 t}\sin\Big(\frac{m\,\pi\,x}{\ell}\Big)\sin\Big(\frac{n\,\pi\,y}{h}\Big),\quad m,n=1,2,3,\dots .$$

$$u(t,x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} b_{mn}\,e^{-k\left(\frac{m^2}{\ell^2}+\frac{n^2}{h^2}\right)\pi^2 t}\sin\Big(\frac{m\,\pi\,x}{\ell}\Big)\sin\Big(\frac{n\,\pi\,y}{h}\Big) \tag{5}$$

Section 12.3, page 502, line −1

$$T_{mn}(t) = e^{-k\left(\frac{m^2}{\ell^2}+\frac{n^2}{h^2}\right)\pi^2 t},\quad m,n=0,1,2,\dots .$$

Section 12.3, page 509, Problem 13

13. Solve (**) in Problem 12 directly by…

Section 12.3, page 509, Problem 14

Last equation should be $u(0,x,y) = \sin 3x\,\sin y + \sin x \sinh(\pi - y)\,\mathrm{csch}\pi,\quad 0 \le x \le \pi,\ 0 \le y \le \pi.$

Hints and Answers

page 594

Section 2.1

9. $(1 + C\,e^{-t})\cos t$

11. $\sqrt{t}\,(t + C)$

21. $C\,e^{t} - t - 1$

Section 2.2.1

1. 18.7 s, −82.8 m/s

3. (b) 131 m

Section 2.2.2

3. $A/40 = e^{-.05t}$, 59.9 minutes

page 597

Section 3.5.2

1. (a) *second line:* $y = \begin{cases} (316 - 1.40\,t)^{2/5}, & 0 \le t \le 226 \\ 0, & t > 226 \end{cases}$

page 601

Section 5.1

11. $(4 - 0.01t)\,y'' + 2y' + y = -0.001$

13. $(m_0 - \rho\,t)\,y'' + k\,y = T$

page 605

Section 6.2

7. $-\frac{1}{4}\,(t^2+t)\,e^{-t}$

Section 6.4

7. (a) $-\frac{1}{2}\,t\,(\cos t - \sin t)$
(b) $-\frac{1}{2}\,t\,(3\cos t + \sin t)$

Section 6.5

7. *second line:* $= \frac{1}{2\omega}\,e^{-pt/2}\sqrt{4\omega^2 y_0^2 + \dots}\ \dots$

page 606

Section 6.6

3. $\frac{9}{16}(2\cos t - \cos 5t)$

page 608

Section 7.4

5. *second line:* $F_0(s) = \frac{1-e^{-s}}{s^2} - \frac{e^{-s}}{s}$
third line: $F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s\,(1-e^{-s})}$

11. $-t + \dfrac{1}{2\pi}\sin(2\pi t) + (1-\cos(2\pi t))\displaystyle\sum_{n=0}^{\infty} h(t-n)$

page 612

Section 8.5

33. *second line:* $Y(t) = e^{-t}\begin{pmatrix} 1 & 2t & 2+4t^2 \\ 0 & 1 & 1+4t \\ 1 & 2 & 4t^2 \end{pmatrix}$

pages 617, 618

Section 9.4

7. *fourth line:* $\left\{(x,y) \mid x \geq 0,\ 0 \leq y \leq \frac{9}{4}\,x + 5,\ x+y \leq \frac{33}{4}\right\}$
sixth line: $y' \leq \frac{9}{4y}$ if $y > 0$;
seventh line: $x' \geq 1 - 4/y$ if $y > 0$;
eighth line: $\frac{dy}{dx} \leq 9/4$ if $y \geq 5$.

page 620

Section 10.5

2. *Hint:* Let $P(t)$ be the position of the bob in rectangular coordinates. Also, the potential energy …

pages 624, 625

Section 11.5

3. *first line:* $\lambda_0 = 0$, $\lambda_n = -\frac{1}{4} - (n-1)^2\pi^2$, $n \geq 1$

15. (c) *Each* λ *in the displayed equation should be* λ_n*. (Only in part (c)!)*

page 628

Section 12.2

13. *last line:* $a_n = \frac{2}{\ell}\int_0^\ell \ldots$

15. *last line:* $a_n = \frac{2}{\ell}\int_0^\ell \ldots$

Section 12.3

9. *second line: The* x *before* $\sin ny$ *should be a times sign.*

Back endpapers, right side, line 13

$$t/k - \lfloor t/k \rfloor \qquad\qquad \frac{1}{k\,s^2} + \frac{1}{s\,(1 - e^{ks})}$$