1. A can of soda is taken from the refrigerator with a temperature of 3°C. The temperature of the room is 23°C. If the temperature of the can of soda has risen to 8°C after 10 minutes, what will be its temperature after 30 minutes?

\[
T' = -k(T - 23), \quad T(0) = 3
\]

\[
T(t) = 23 + Ce^{-kt}, \quad T(0) = 3 \Rightarrow C = -20
\]

\[
T(t) = 23 - 20e^{-kt}
\]

\[
T(10) = 8 \Rightarrow e^{-10k} = 15/20 = 3/4 \Rightarrow k = -0.1 \ln(3/4) \approx 0.0288
\]

\[
T(t) \approx 23 - 20e^{-0.0288t}
\]

\[
T(30) \approx 23 - 20e^{-0.0288(30)} \approx 14.6^\circ C
\]

2. A 10 gallon tank is initially full of pure water. At time \( t = 0 \), salt water with a salt concentration of 100 grams/gallon begins to flow in at a steady rate of \( \frac{1}{2} \) gallon/minute. The fluid in the tank is kept well-mixed and flows out at the same rate. Set up and solve the initial-value problem for the amount \( A(t) \) of salt in the tank at time \( t \) minutes.

\[
A' = 100 \times \frac{1}{2} - \frac{A}{10} \times \frac{1}{2}, \quad A(0) = 0
\]

\[
A' + \frac{1}{20} A = 50, \quad A(0) = 0
\]

\[
A = 1000 + Ce^{-t/20}, \quad A(0) = 0 \Rightarrow C = -1000
\]

\[
A = 1000 (1 - e^{-t/20})
\]

3. Use an integrating factor to find the general solution of \( y' + \frac{1}{t+1} y = 6t \).

\[
e^{\int \frac{1}{t+1} dt} = e^{\ln(t+1)} = t + 1
\]

\[
((t+1)y) = (t+1)6t = 6t^2 + 6t
\]

\[
(t+1)y = 2t^3 + 3t^2 + C
\]

\[
y = \frac{2t^3 + 3t^2 + C}{t+1}
\]
4. Find a linear polynomial that satisfies \( y' + 3y = 9t + 3. \)

\[
y = a t + b \Rightarrow y' + 3y = a + 3(a t + b) = 3a t + a + 3b
\]

\[
3a t + a + 3b = 9t + 3 \Rightarrow a = 3, \; b = 0
\]

\[
y = 3t
\]

5. Solve the initial-value problem

\[
y' + y = 2y^3, \; y(0) = 1,
\]

and state the maximal domain of the solution. (Use the substitution \( y = u^m. \))

\[
y = u^m, \; y' = mu^{m-1}u'
\]

\[
mu^{m-1}u' + u^m = 2u^{3m}
\]

\[
u' + \frac{1}{m} u = \frac{2}{m} u^{2m+1}
\]

\[
m = -1/2 \Rightarrow u' - 2u = -4 \Rightarrow u = 2 + Ce^{2t}
\]

\[
y = u^{-1/2} = \frac{1}{\sqrt{2 + Ce^{2t}}}
\]

\[
y(0) = 1 \Rightarrow C = -1
\]

\[
y = \frac{1}{\sqrt{2 - e^{2t}}}, \; -\infty < t < \frac{1}{2} \ln 2
\]

6. Use the substitution \( y = t z \) to find the general solution of \( y' = e^{y/t} + \frac{y}{t}. \)

\[
y = t z, \; y' = z + t z' \Rightarrow z + t z' = e^z + z
\]

\[
e^{-z} dz = \frac{1}{t} dt
\]

\[
-e^{-z} = \ln t + C
\]

\[
z = -\ln(C - \ln t)
\]

\[
y = t z = -t \ln(C - \ln t)
\]
7. The following picture shows the direction field for the equation \( y' = (2 \cos t - y) y \). The isoclines shown correspond to slopes of \( y' = -3, -1, 0, \frac{1}{2} \).

(a) Sketch the graph of the solution satisfying \( y(-2) = 1 \).

(b) Sketch the graph of the solution satisfying \( y(-1) = 2 \).