Math 4011 Advanced Calculus I

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Textbook: *Elementary Real and Complex Analysis*, Georgi E. Shilov, Dover.

Grading Policy: There will be (roughly) weekly homework assignments, which will be added below to this document and posted at www.math.armstrong.edu/faculty/holllis/classes/4011. These will comprise 50% of your grade. Three in-class tests will count 10% each, and a final exam will fill the remaining 20%. The tests and final *may* include take-home portions.

Chapter 1 The Real Numbers

We will move very quickly through the axioms that define $\mathbb{R}$ and their consequences, with the exception of §§1.6–1.8, which are crucial. However, please read §§1.1–1.5 thoroughly. Work all of the problems at the end of the chapter.

Assignment 1: Problems 6–9. Due Friday, August 31.

Assignment 2: Problems 18,19. Due Monday, Sept 10.

Chapter 2 Sets

The important sections here are §§2.2–2.4, 2.8. Work all of the problems at the end of the chapter.

Assignment 3: Problems 5,6. Due Friday, Sept 14.

Chapter 3 Metric Spaces

Subsections to skip: 3.14–3.16, 3.34. Do problems 4, 5, 6, 8.

Assignment 4: Let $\mathcal{C}[0,1]$ denote the space of continuous functions on the interval $[0,1]$ with the metric

$$\rho(f,g) = \max_{0\le x\le 1} |f(x) - g(x)|.$$

(a) Find $\rho(x^2, x^3)$.

(b) Sketch the graph of $f(x) = \sin(2\pi x)$ along with a generic function contained in the open ball centered at f with radius 1/4.

(c) Construct a function contained in the *sphere* centered at $f(x) = \sin(2\pi x)$ with radius 1/10, i.e., a function g such that $\rho(f,g) = 1/10$.

(d) Show that the sequence $\{\cos(2\pi x/n)\}_{n=1}^{\infty}$ converges to the constant function $f(x) = 1$.

(e) Let $A = \{f_1, f_2, f_3, \ldots\}$, where

$$f_n(x) = \begin{cases} 1 & \text{if } 0 \le x \le \frac{1}{n+1}, \\ (n+1)(1-nx) & \text{if } \frac{1}{n+1} < x \le \frac{1}{n}, \\ 0 & \text{if } \frac{1}{n} < x \le 1. \end{cases}$$

Sketch the graphs of f_3 and f_4. Then show that A has no limit points.

Assignment 5: Recall the notation:

$$\text{int}A = \{\text{interior points of } A\},\ A' = \{\text{limit points of } A\},\ \overline{A} = A \cup A',\ \partial A = \overline{A} \cap \overline{A^c}.$$

1. Give an example of $A \subset \mathbb{R}$ for which
   (a) $\overline{A} \ne \overline{\text{int}A}$ (b) $\text{int}A \ne \text{int}\overline{A}$ (c) $\overline{A} = \mathbb{R}$ and $\text{int}A = \emptyset$
2. Prove that $\partial A = \emptyset$ if and only if A is both open and closed.
3. Prove that A' is closed.
4. Prove that $A \cup \partial A = \overline{A}$.

Chapter 5 Continuous Functions

Sections to skip: 5.5–5.8. Work problems 5–10, 12, 14, 18.

Assignment 6: Problems 14 and 18.

Chapter 6

Chapter 7