

# FLIGHT OF A PARATROOPER

**Goal:** In this project your group will analyze the flight of a paratrooper as he/she jumps from a plane with a parachute. A skydiver or paratrooper depends on the air resistance provided by the parachute to soften the landing. Parachutes have a constant  $\rho$  associated with them called the **resistance coefficient**. In this project your team will use mathematical models in order to choose an appropriate resistance coefficient.

## Names and Primary Responsibilities of Group Members:

**Main Computations Analyst:**

**Geometer:**

**Recorder:**

**Technology Operator:**

The following two equations give the velocity and distance from the ground, respectively, of a paratrooper who jumps from a plane at a height of  $y_o$  feet and pulls the chute immediately. Both acceleration due to gravity and deceleration due to air resistance are taken into consideration in this model. In these equations,  $v_o$  and  $v_T$  denote the initial and terminal velocity of the paratrooper.

$$v(t) = \left(v_o + \frac{32}{\rho}\right)e^{-\rho t} - \frac{32}{\rho}$$

$$y(t) = y_o + v_T t + \frac{1}{\rho}(v_o - v_T)(1 - e^{-\rho t})$$

**Modeling Stage:** In this stage the *Computations Analyst* must guide the team in constructing the specific set of two equations which model the distance and velocity of a paratrooper who jumps from an altitude of 25000 feet (this is called a “*Halo Jump*”) with a parachute carrying a resistance coefficient of 2.25.

1. Determine the values of the constants  $v_o$  and  $y_o$ .
2. One way of computing the terminal velocity of the paratrooper is to determine when he/she will hit the ground and then compute the velocity at that time. Consider this method and explain one possible hindrance to using this method.
3. Another way defining the terminal velocity is to compute the limiting velocity, that is,  $v_T = \lim_{t \rightarrow \infty} v(t)$ . Compute  $v_T$  using this method and use this value of  $v_T$  in the model.

Record the specific distance and velocity models here:

$$v(t) =$$

$$y(t) =$$

**Analysis Stage:** In this stage your team will use the constructed model to analyze the actual flight of the paratrooper who jumps from 25000 feet.

1. How far did this paratrooper fall during the first 5 seconds? 20 seconds? during the first 10 minutes?
2. Do you detect a flaw in this mathematical model? Use a piecewise defined function to construct a more accurate function  $y(t)$ . this flaw.
3. What was this paratrooper's velocity during the first 5 seconds? 20 seconds?
4. Determine the acceleration function of our paratrooper. With the aid of a graphing utility, plot it below and explain the physical significance of the curve.

**Editorial Stage:** In a newspaper report, a paratrooper was said to have jumped out of a plane flying at 10000 feet and to have landed at a speed of 100 miles per hour. Allegedly she hit the ground after 20 seconds. What must have been the resistance coefficient of this parachute? Led by the team's *recorder*, write a short article for the newspaper which retracts the flawed story. Include any relevant data.