Representations of Infinite Monomial Groups

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Abstract

In recent years the representation theory of linear algebraic groups has been given new value by its connection to the field of differential algebra. In particular, representation theory of linear algebraic groups has direct applications to the computation of differential Galois groups as well as the factorization of linear differential equations. In general, effective algorithms for decomposing representations of infinite linear algebraic groups of degree larger than four are not readily available in the literature. In this paper we show that an infinite monomial linear algebraic group of degree \( n \) must have a very specific form which can be completely determined by its dimension as an algebraic variety as well as its permutation action on the standard representation, \( V \). Furthermore this representation of the group allows us to apply the procedures found in (Hessinger 2002) for decomposing representations of infinite monomial groups of arbitrary degree.

1 Introduction

A linear algebraic group \( G \) of a degree \( n \) can be classified via its action on its standard representation, \( V \cong \mathbb{C}^n \). The group \( G \) is either reducible, in which case \( V \) has a nontrivial \( G \)-invariant subspace, irreducible and imprimitive, in which case \( V \) can be written as a direct sum of subspaces which are permuted by \( G \), or irreducible and primitive. Imprimitive groups which permute subspaces \( V_i \) of dimension one are called monomial groups. This set of one dimensional subspaces form the system of imprimitivity for \( G \). In light of our interest in decomposing representations in order to contribute to factoring differential equations and computing differential Galois groups, we will restrict our focus to infinite monomomial groups which are unimodular, that is subgroups of \( \text{SL}(n, \mathbb{C}) \).

2 Results in Progress

Theorem 2.0.1 Let \( G \) be an infinite monomial unimodular linear algebraic group of degree \( n \). Let \( \mathcal{P} \) be the permutation group \( \mathcal{P}(G) \) giving the action of \( G \) on its system of imprimitivity. Let \( H = \text{Ker}(\mathcal{P}) \). If \( \mathcal{P} \) is primitive, then the connected component \( H^o \) of \( H \) is equal to \( \text{diag}(\text{SL}(n, \mathbb{C})) \).

Theorem 2.0.2 Let \( H^o \) be the connected component of the diagonal subgroup of a degree \( n \) unimodular monomial group \( G \) and let \( \mathcal{P} \) be the permutation action of \( G \). Then \( H^o \) is of the form

\[
\text{diag}(\text{SL}(k, \mathbb{C}))_1 \times \text{diag}(\text{SL}(k, \mathbb{C}))_2 \times \ldots \times \text{diag}(\text{SL}(k, \mathbb{C}))_m
\]

where \( k \) is a minimal block size for the permutation group \( \mathcal{P} \) and \( n = mk \). Furthermore, these \( m \) copies of \( \text{diag}(\text{sl}k) \) may or may not be distinct depending upon dimension of the group \( H^o \) as an algebraic variety.