1189. *Proposed by Peter A. Lindstrom, Batavia, NY.*

If F_n denotes the nth Fibonacci number, show that $F_{4n+2} - (2n+1)$ is divisible by 5.

Solution by the **Armstrong Problem Solvers***, Armstrong Atlantic State University, Savannah, GA.*

Notice that the Fibonacci sequence modulo 5 is periodic with period 20, since $F_{20} \equiv 0$ and $F_{21} \equiv 1$ modulo 5. Thus, if $n \equiv m \pmod{20}$, then $F_n \equiv F_m \pmod{5}$. For integers r with $0 \le r \le 4$, we compute F_{4r+2} to be congruent modulo 5 to 1, 3, 0, 2, and 4, respectively, so $F_{4r+2} \equiv 2r+1 \pmod{5}$ for $0 \le r \le 4$. If n is any nonnegative integer, then we can write $n = 5q + r$, where q and r are nonnegative integers and $0 \le r \le 4$. Then $4n + 2 = 20q + 4r + 2 \equiv 4r + 2 \pmod{20}$, so that $F_{4n+2} \equiv F_{4r+2} \equiv 2r + 1 \equiv 2n + 1 \pmod{5}$.

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