1187. *Proposed by Brian Bradie, Christopher Newport University.*
Evaluate
$$\int_0^1 \frac{\ln(1+x)}{x}\,dx.$$

Solution by the **Armstrong Problem Solvers**, *Armstrong Atlantic State University, Savannah, GA.*

$$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}.$$

For $-1 < x \le 1$,

$$\begin{aligned}
\ln(1+x) &= \int_0^x \frac{1}{1+t}dt \\
&= \int_0^x \sum_{n=0}^{\infty}(-1)^n t^n dt \\
&= \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1} \\
\frac{\ln(1+x)}{x} &= \sum_{n=0}^{\infty}(-1)^n \frac{x^n}{n+1} \\
\int \frac{\ln(1+x)}{x}dx &= C + \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{(n+1)^2} \\
\int_0^1 \frac{\ln(1+x)}{x}dx &= \sum_{n=0}^{\infty}(-1)^n \frac{1}{(n+1)^2} \\
&= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}.
\end{aligned}$$

If we call this alternating sum S, then since $\sum_{n=1}^{\infty}\frac{1}{n^2} = \Gamma(2) = \frac{\pi^2}{6}$, then

$$\begin{aligned}
\frac{\pi^2}{6} - S &= \sum_{n=1}^{\infty}\left(\frac{1}{n^2} - \frac{(-1)^{n+1}}{n^2}\right) \\
&= 2\sum_{n=1}^{\infty}\frac{1}{(2n)^2} \\
&= \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^2} \\
&= \frac{\pi^2}{12},
\end{aligned}$$

and $S = \frac{\pi^2}{6} - \frac{\pi^2}{12} = \frac{\pi^2}{12}$.

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