MM 1816. *Proposed by Mehmet Sahin, Ankara University of Science, Ankara, Turkey.*

Let ABC be a triangle with $a = BC$, $b = AC$, and $c = AB$. Let $A'B'C'$ be another triangle with $B'C' = \sqrt{a}$, $C'A' = \sqrt{b}$, and $A'B' = \sqrt{c}$. Prove that

$$\sin\left(\frac{1}{2}A\right)\sin\left(\frac{1}{2}B\right)\sin\left(\frac{1}{2}C\right) = \cos A' \cos B' \cos C'.$$

Solution by the **Armstrong Problem Solvers**, *Armstrong Atlantic State University, Savannah, GA.*

Using the half-angle formula for sine and the law of cosines,

$$\begin{aligned}
\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} &= \sqrt{\frac{1-\cos A}{2}}\sqrt{\frac{1-\cos B}{2}}\sqrt{\frac{1-\cos C}{2}} \\
&= \sqrt{\frac{2bc-b^2-c^2+a^2}{4bc}\cdot\frac{2ca-c^2-a^2+b^2}{4ca}\cdot\frac{2ab-a^2-b^2+c^2}{4ab}} \\
&= \sqrt{\frac{(a^2-(b-c)^2)(b^2-(c-a)^2)(c^2-(a-b)^2)}{64a^2b^2c^2}} \\
&= \frac{\sqrt{(a+b-c)(a-b+c)(b+c-a)(b-c+a)(c+a-b)(c-a+b)}}{8abc} \\
&= \frac{(b+c-a)(c+a-b)(a+b-c)}{8abc} \\
&= \frac{b+c-a}{2\sqrt{bc}}\cdot\frac{c+a-b}{2\sqrt{ca}}\cdot\frac{a+b-c}{2\sqrt{ab}} \\
&= \cos A' \cos B' \cos C'.
\end{aligned}$$

Armstrong Problem Solvers
Department of Mathematics
Armstrong Atlantic State University
11935 Abercorn Street
Savannah, GA 31419-1997
e-mail: James.Brawner@armstrong.edu