MM 1807. *Proposed by Lenny Jones, Shippensburg University, Shippensburg, PA.*

Let P be a polynomial with integer coefficients and let s be an integer such that for some positive integer n, $s^{n+1}P(s)^n$ is a positive zero of P. Prove that $P(2) = 0$.

Solution by the **Armstrong Problem Solvers**, *Armstrong Atlantic State University, Savannah, GA.*

Since $s^{n+1}P(s)^n$ is a zero of P, then $P(x) = \left(x - s^{n+1}P(s)^n\right) Q(x)$, where $Q(x)$ is a polynomial with integer coefficients, and $P(s) = \left(s - s^{n+1}P(s)^n\right) Q(s)$. If we let $m = sP(s)$, then $m = s^2(1 - m^n)Q(s)$, where $Q(s)$ is an integer. Notice that $m \neq 1$, since otherwise the previous equation would give $1 = 0$. Since $1 = m^{n-1}m + 1 - m^n$, then m and $1 - m^n$ must be relatively prime. Yet $\frac{m}{1-m^n} = s^2Q(s)$ is an integer, so $1 - m^n = \pm 1$, and $m^n = 0$ or 2. If $m = 0$, then $sP(s) = 0$, so either $s = 0$ or $P(s) = 0$, contradicting the assumption that $s^{n+1}P(s)^n > 0$. Thus, $m^n = 2$, which means $m = sP(s) = 2$ and $n = 1$. Since $s^2P(s) > 0$, either $s = 2$ and $P(s) = 1$, or $s = 1$ and $P(s) = 2$. If $s = 2$, then since $m = s^2(1 - m^n)Q(s)$, we would have $2 = 4(-1)Q(s)$, which contradicts $Q(s)$ being an integer. Thus, $s = 1$, $P(s) = 2$, and $s^{n+1}P(s) = 2$ is a zero of P.

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