MH 230, A Plane Slice. What shape must be removed from the plane in order that the sculpture can rotate freely?

Solution by the **Armstrong Problem Solvers**, *Armstrong Atlantic State University, Savannah, GA.*

If $r \neq |a|$, then the shape must be a hyperbola; if $r = |a|$, then it is a pair of intersecting lines.

Parametric equations for the inclined rod are given by $x = r\cos\theta - t\sin\theta$, $y = r\sin\theta + t\cos\theta$, and $z = t\tan\alpha$. The plane $y = a \neq 0$ intersects the rod where $r\sin\theta + t\cos\theta = a$, or $t = a\sec\theta - r\tan\theta$. Substituting for t gives

$$x = r\cos\theta - a\tan\theta + r\sin\theta\tan\theta = r\sec\theta - a\tan\theta$$

and $z = \tan\alpha(a\sec\theta - r\tan\theta)$. We compute

$$\begin{aligned} x^2 - \frac{z^2}{\tan^2\alpha} &= (r^2 - a^2)\sec^2\theta - (r^2 - a^2)\tan^2\theta \\ &= (r^2 - a^2)(\sec^2\theta - \tan^2\theta) \\ &= r^2 - a^2. \end{aligned}$$

If $r > |a|$, then the curve removed will be both branches of a hyperbola opening horizontally, along with a horizontal slit of length $2\sqrt{r^2 - a^2}$ to accommodate the horizontal bar. If $r < |a|$, then the curve removed will be both branches of a hyperbola opening vertically. If $r = |a|$, then we must remove two lines with equations $z = \pm x\tan\alpha$.

Armstrong Problem Solvers
Department of Mathematics
Armstrong Atlantic State University
Savannah, GA 31419-1997
e-mail: James.Brawner@armstrong.edu