MH 224. *Proposed by Steven Tedford, Misericordia University.* Answer any of the following three questions about the Squeezed Chip Puzzle:

(a) Is $s(7) \leq 10$? That is, can a total of 15 chips be sorted in at most 10 moves?

(b) Can you find an upper bound function $g(n)$ such that $s(n) \leq g(n)$ for all n?

(c) Can you find a simple formula for $s(n)$?

Solution to (a) and (b) by the **Armstrong Problem Solvers**, *Armstrong Atlantic State University, Savannah, GA.*

Let Li denote the move that consists of moving the ith and $(i+1)$st chips (from left to right) to the left of the row and squeezing shut any gap, where $1 \leq i \leq 2n$. Similarly, let Ri denote the move that consists of moving the same two chips to the right of the row and squeezing shut any gap.

We can answer part (a) in the affirmative by exhibiting the following sequence of ten moves: R1, R2, R1, R2, R1, R2, R1, R3, R5, R7.

(b) Let $g(n) = \lfloor \frac{3n-1}{2} \rfloor$. We show that $s(n) \leq g(n)$ for all natural numbers n by exhibiting a sequence of $g(n)$ moves that sorts the chips. We give different algorithms depending on the parity of n. If n is odd, then $n = 2k-1$, where k is a natural number. Repeat the combination [R1, R2] $k-1$ times and then do one more $R1$ move to get the chips in the following order:

$$\underbrace{RRBB}_{k-1 \text{ times}} \; RRB$$

Then the following sequence of moves will move each of the $k-1$ adjacent pairs of blue chips to the right end: [R3, R5, R7, ..., Rn]. This sequence has a total of $2(k-1)+1+(k-1) = (3n-1)/2$ moves.

If n is even, then $n = 2k$, where k is a natural number. Repeat the combination [R1, R2] $k-1$ times and then do one more $R1$ move to get the chips in the following order:

$$RB \; \underbrace{RRBB}_{k-1 \text{ times}} \; RRB$$

Then the following sequence of moves will move each of the k adjacent pairs of red chips to the left end: [L3, L7, L11, ..., L$(4k-1)$]. This sequence has a total of $2(k-1)+1+k = (3n-2)/2$ moves. Thus, in either case a sequence of $g(n) = \lfloor \frac{3n-1}{2} \rfloor$ moves is sufficient to sort the chips, and $s(n) \leq g(n)$ for all n.

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