South**E**ast **R**egional **M**eeting **O**n **N**umbers

Armstrong Altantic State Univ.
April 16-17, 2011

Schedule of Talks

Saturday, April 16 (University Hall 156)

9:00am	Coffee
9:30-10:00	Michael Mossinghoff, Davison College, *Extremal polygons, and roots of unity.*
10:00-10:30	Break
10:30-11:30	Jim Coykendall, North Dakota State Univ., *Factorization of Ideals, Elements, and Norms in Rings of Algebraic Integers.*
11:30-1:00	Lunch Break
1:00-1:30	Mark Budden, West Carolina Univ., *Quartic Residue Graphs and Jacobi Sums*
1:40-2:10	Tingyao Xiong, Radford Univ., *General Eulerian Polynomials, Eulerian Numbers and Triangular Arrays*
2:10-2:40	Break
2:40-3:10	Jenny Fuselier, High Point Univ., *Traces of Hecke Operators in Level 1 and Gaussian Hypergeometric Functions*
3:20-3:50	Jeff Beyerl, Clemson Univ., *Factorizations of Eigenforms and Maeda's conjecture*
4:00 - 4:30	John Webb, Univ. of South Carolina, *Hecke nilpotency and 2^t-core partitions*

Sunday, April 17 (University Hall 156)

8:30am	Coffee
9:00-9:30	Zachary A. Kent, Emory Univ., *Hida's Control Theorem*
9:50-10:50	Daniel Kane, Harvard Univ., *Ranks of 2-Selmer Groups of Twists of an Elliptic Curve*
11:10-11:40	Hui Xue, Clemson Univ., *The derivative of an Eisenstein series*

List of Abstracts

Plenary Speakers

Speaker: Jim Coykendall, North Dakota State University

Title: Factorization of Ideals, Elements, and Norms in Rings of Algebraic Integers.

Abstract: In this talk we will explore factorization in rings of algebraic integers. Rings of algebraic integers are the playground of algebraic number theory and may be considered as a natural generalization of the "ordinary" integers. Unfortunately (or fortunately if your tastes in factorization tend to the exotic) rings of algebraic integers do not necessarily have unique factorization (if they did, the infamous Fermat's Last Theorem would have been proved more than a century ago).

Rings of algebraic integers do, however, possess the "next best thing" to unique factorization of elements: their ideals have unique factorization into prime ideals. This property, coupled with the fact that prime ideals in rings of algebraic integers are distributed more or less evenly into ideal classes, allow us to glean interesting and strong results concerning factorization of ordinary elements into irreducibles.

This talk will focus on techniques using both unique factorization of ideals and the usual norm function to illustrate factorization properties of elements in rings of algebraic integers. We will show, among other things, that unique factorization in rings of integers is determined by properties of the norm (in the Galois case) and we will show how to extract the elasticity (a natural measure of how "bad" factorizations can be) based on the class group of the ring of algebraic integers. Many examples will be highlighted to excite the imagination and hopefully increase intuitive understanding.

Speaker: Daniel Kane, Harvard University

Title: Ranks of 2-Selmer Groups of Twists of an Elliptic Curve

Abstract: Let $E/\mathbb{Q}$ be an elliptic curve with full 2-torsion over $\mathbb{Q}$. We wish to study the distribution of the ranks of the 2-Selmer groups of twists of E as we vary the twist parameter. A recent result of Swinnerton-Dyer shows that if E has no cyclic 4-isogeny defined over $\mathbb{Q}$, then the density of twists with given rank approaches a particular distribution. Unfortunately Swinnerton-Dyer used an unusual notion of density essentially given as the number of primes dividing the twist parameter goes to infinity. We extend this result to cover density in the natural sense.

Contributed Speakers

Speaker: Jeff Beyerl, Clemson University

Title: Factorizations of Eigenforms and Maeda's conjecture

Abstract: In this talk we will discuss factorizations of cuspidal eigenforms and implications of a conjecture of Maeda in this direction.

Speaker: Mark Budden, West Carolina University

Title: Quartic Residue Graphs and Jacobi Sums

Abstract: One interesting class of graphs with ties to classical topics in number theory are Paley Graphs. They are defined as having vertices in the finite field $\mathbb{F}_q$, where $q \equiv 1 \pmod 4$, and vertices a and b are adjacent if and only if $a-b$ is a quadratic residue modulo q. It is natural to extend this definition to higher powered residues. In this talk, we will discuss some of the basic properties of quartic residue graphs and give an explicit enumeration of the number of triangles contained in such graphs. To arrive at our main result, it will be necessary to exploit known computations of quartic Jacobi sums.

Speaker: Jenny Fuselier, High Point University

Title: Traces of Hecke Operators in Level 1 and Gaussian Hypergeometric Functions

Abstract: In this talk, we explore relationships between traces of Hecke operators, counting $\mathbb{F}_p$-points on families of elliptic curves, and values of Gaussian hypergeometric functions. We begin by introducing these hypergeometric functions, then give a survey of results linking their values to traces of Hecke operators in levels 8, 4, and 2. We then focus on how traces of Hecke operators in level 1 relate to values of a Gaussian ${}_2F_1$ function. In particular, for primes p congruent to 1 mod 12, we present a formula for Ramanujan's τ-function at p in terms of 10^{th} powers of a hypergeometric function.

Speaker: Zachary A. Kent, Emory University

Title: Hida's Control Theorem

Abstract: Motivating with a classical conjecture of Atkin, we introduce the beautiful Control Theory of Hida for integer weight p-ordinary modular forms. A new proof will be given for an effective version of the Control Theorem.

Speaker: Michael Mossinghoff, Davidson College

Title: Extremal polygons, and roots of unity.

Abstract: For a positive integer n that is not a power of 2, precisely the same family of convex polygons with n sides is optimal in three different extremal problems involving the perimeter, the diameter, and the width of an n-gon. For example, these shapes have maximal perimeter relative to their diameter. We investigate the combinatorial question of determining the number of different convex n-gons $E(n)$ that are optimal in these isodiametric and isoperimetric problems. We first study the extremal polygons that exhibit a particular periodic structure, and then turn to the question of the existence of sporadic extremal shapes, which appear to occur only for certain values of n. We obtain a lower bound on $E(n)$ that depends on the factorization of n, and prove that it is an exact formula in certain cases. We also show that these geometric questions can be recast as problems about sums of certain roots of unity, and present some open problems.

Speaker: John Webb, University of South Carolina

Title: Hecke nilpotency and 2^t-core partitions

Abstract: We say that a partition of n is t-core if none of the hook lengths in the Ferrers-Young diagram are divisible by t. We prove a wealth of new congruences for 2^t-core partitions modulo 2 by examining the nilpotency of Hecke operators on level 1 cusp forms. Based on extensive calculations we give a conjecture for the structure of T_p-cyclic subspaces for these forms.

Speaker: Tingyao Xiong, Radford University

Title: General Eulerian Polynomials, Eulerian Numbers and Triangular Arrays

Abstract: The Eulerian Polynomials have been introduced by Euler himself back in 1755. The combinatorial meanings of Eulerian numbers which are closely related to Eulerian

Polynomials have been discovered by Riordan two hundred years later. The defnitions and properties of Eulerian polynomials and Eulerian numbers have been thoroughly studies and extended in both directions of analytical number theory and combinatorics. In this paper, we will generalize the defnition of Eulerian polynomials and Eulerian numbers to general arithmetic progressions. Under our defnition, we have been successful in generalizing many well-known properties of traditional Eulerian polynomials and Eulerian numbers.

Speaker: Hui Xue, Clemson University

Title: The derivative of an Eisenstein series

Abstract: In this talk we first construct an Eisenstein series from the quadratic space associated to an imaginary quadratic field. We then study the central derivative of the Eisenstein series and show that its Fourier coefficients are given by degrees of certain arithmetic cycles.

Organizers

Sungkon Chang
Joshua Lambert