

Putnam Seminar 2008: Take-Home Test I
Due by Oct 1

Directions/Comments:

The focus point of grading this take-home test will be the mathematical writing skill. For full credits, the graders will expect solutions in thorough, well-written, elaborate writing. Some of the principles of writing good solutions would be helping the graders to fully understand examinee's solutions, and helping to not raise doubts to the graders that the examinee might not know or think about certain details, cases/possibilities, or the correctness of his/her reasoning.

The problems #1–3 are either similar or equal to the problems we did in class. For these three problems, do not take any part of solutions for granted, but write self-contained, thorough solutions for full credits.

Tests will be independently graded by both Dr. Brawner and Dr. Chang.

An A will be assigned if both graders give A's.
A B will be assigned if no graders give an F and at least one grader assigns a B.
A C will be assigned if both graders give a C.
An F will be assigned if one grader gives an F.

A C-level question is #1, B-level questions are #2 & 3, and A-level questions are #4 & 5. The questions #4 & 5 are A-leveled since their solutions were not discussed during the seminar, and they are not necessarily more difficult than the lower-level questions. Each test will be graded by a “top-down” method:

An A will be assigned if at least one A-level and all other level questions have good solutions.
A B will be assigned if at least one B-level and the C-level questions have good solutions, but no A-level questions have good solutions.
A C will be assigned if the C-level question has a good solution, but no B-level questions have good solutions.
An F will be assigned if the C-level question does not have a good solution.

Examinees are NOT allowed to discuss about the test problems with anyone, prior to submission, or to use any sources other than our course website, to write solutions. You have only one chance to submit your solutions; please, speak to the instructors, before the due date, if there are questions or concerns about this guide line for the test.

Please print solutions dark enough for clear photocopy.

Putnam Seminar 2008: Take-Home Test I Problems

Please, read the Directions/Comments before writing up solutions.

#1 Prove or disprove that there are three distinct prime numbers p , q , and r such that the following is rational:

$$\sqrt{p} + \sqrt{q} + \sqrt{r}.$$

#2 Finitely many lines divide the plane into regions. Show that these regions can be colored by two colors in such a way that neighboring regions have different colors.

#3 Prove that for any positive integer $n \geq 2$ there is a positive integer m that can be written simultaneously as a sum of $2, 3, \dots, n$ squares of nonzero integers. Find a positive integer m which can be written simultaneously as a sum of $2, 3, \dots, 7$ squares of nonzero integers, and write the six sums.

#4 Prove that any positive integer can be represented as $\pm 1^2 \pm 2^2 \pm \dots \pm n^2$ for some positive integer n and some choice of the signs.

#5 Given nine points inside the unit square, prove that some three of them form a triangle whose area does not exceed $1/8$.