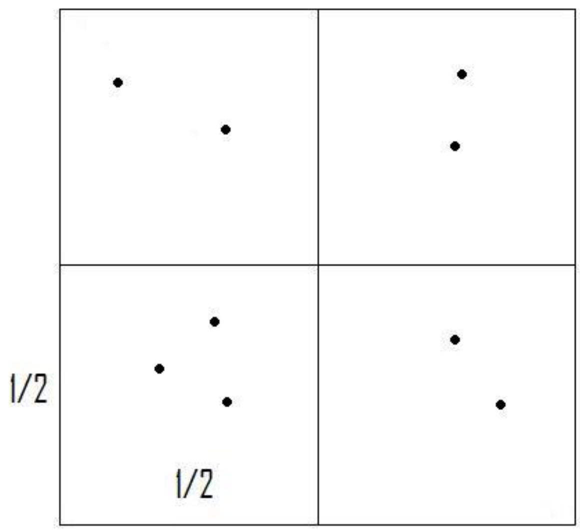
The Problem:

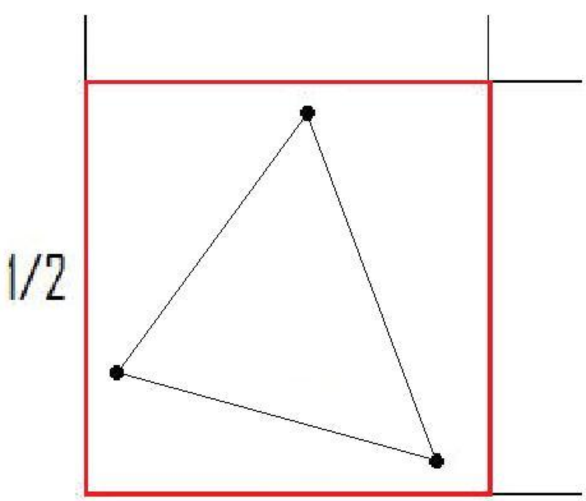
Given nine points inside the unit square, prove that some three of them form a triangle whose area does not exceed 1/8.

The solution:

You can take the square and divide it into four areas by bisecting each side with a line as shown below:



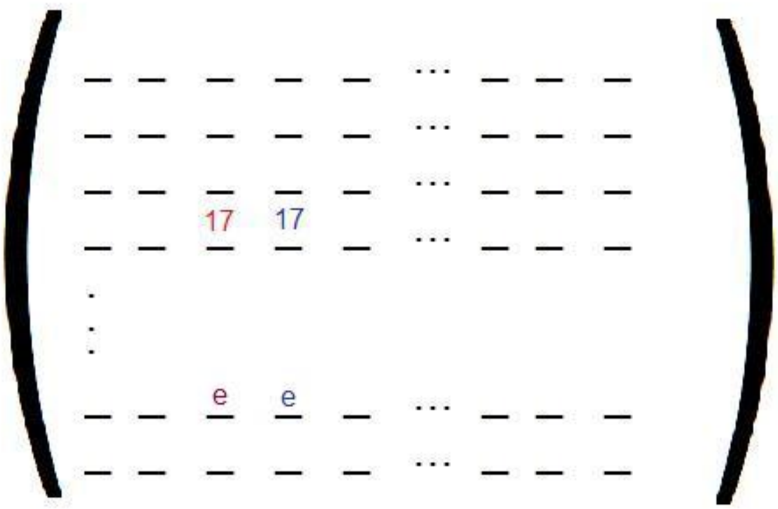
Now that the Square is divided into areas we can apply the Pigeon Hole Principal. Because 9 = 2*4 + 1 we can say that at least one of our four regions contains 2 +1 or 3 points. So we have three points contained in a ½ units by ½ units area. Thus the triangle formed by these three points will have a base and height no larger than ½. Therefore the area of such a triangle will have an upper bound of ½ * ½ * ½ or 1/8.



The Problem:

Alan and Barbara play a game in which they take turns filling entries of an initially empty 2008 X 2008 array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?

The Solution:



Barbara has an advantage, though it might not be clear at first glance. If the determinant of the final matrix is zero then several other things are also true. The particularly important concept is that the rows and columns of the final matrix form a linearly independent set of vectors. This is what leads to Barbara's advantage. Because Barbara goes second her winning strategy is simply to pair up one column with another column and make them identical, thus one will be a linear combination of the other forcing the vectors to be linearly dependent. Whenever Alan enters the first real number, she can go to the adjacent column and make the same entry in the same row, pairing the columns. The only other thing that needs to be address is whether or not Alan can cause Barbara to make a leading entry in one of the paired columns. However, this is not possible because there are an even number of entries in the 2008 X 2008 array. Provided Barbara doesn't make a leading entry in one of her paired columns she is guaranteed victory in the game.