MATH 2072 FINAL EXAM REVIEW

⋆ Bring a calculator and something to write with.

⋆ Note that this review also serves as a review for the fourth exam, which will only cover sections 9.1-9.5.

Section 6.1 Integration by parts was introduced in this section. Be familiar with the examples that were covered in class. In particular, some examples used integration by parts more than once, some used integration by parts along with the substitution rule, and others required that the resulting equation be solved for the desired integral.

Section 6.2 Be able to evaluate integrals of the forms

$$\int \sin^m x \cos^n x \, dx, \quad \int \tan^m x \sec^n x \, dx, \quad \int \cot^m x \csc^n x \, dx.$$

All trigonometric identities will be given to you, you just need to be able to apply them correctly. Trigonometric substitution was also covered in this section. Such integrals include expressions of the form $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$. Before using trigonometric substitution, always see if regular substitution would work first.

Section 6.3 In this section, we described methods for integrating rational functions. If a rational function is improper, first apply the division algorithm to express it in terms of its quotient and a proper rational function. A proper rational function can be integrated after being expanded as partial fractions. We also looked at the case where an integrand contains a radical and can be rationalized.

Section 6.5 Be able to approximate definite integrals using the Midpoint Rule, Trapezoidal Rule, and Simpson's Rule.

Section 6.6 Two types of improper integrals were defined. Be able to determine when such integrals are convergent or divergent and when they do converge, evaluate them. The Comparison Theorem gave us a way of comparing improper integrals to determine convergence or divergence.

Section 7.2 Be able to find the volume of a solid using the "cross-section" method. Sometimes the solids will be described by rotating a region of the xy-plane about a horizontal or vertical line. Other problems describe a solid that you must position relative to the xy-plane.

Section 7.3 Use the "cylindrical shells" method to find the volume of a solid. You may be given a solid in which only one of these two methods work.

Section 7.4 Given a smooth function $y = f(x)$ (f' is continuous), find the length of the curve over the interval $a \leq x \leq b$:

$$\text{Arc Length} = \int_a^b \sqrt{1 + (f'(x))^2}\; dx.$$

Of course, the roles of x and y may be switched.

Areas of Surfaces of Revolution Find the area of a surface of revolution. If a curve is rotated about the x-axis, use the form $\int 2\pi y\; ds$ and if it is rotated about the y-axis, use the form $\int 2\pi x\; ds$. If the curve is defined by $y = f(x)$, where f is smooth and defined for $a \leq x \leq b$, then the bounds of integration are a and b and

$$ds = \sqrt{1 + (f'(x))^2}\; dx.$$

If the curve is defined by $x = g(y)$, where g is smooth and defined for $c \leq y \leq d$, the bounds of integration are c and d and

$$ds = \sqrt{1 + (g'(y))^2}\; dy.$$

Section 7.5 We began this section by defining *force* and *work* in the case where force is constant. You should be able to work problems using metric units (m, kg, N, J, $\ldots$). Work was also defined when force can be described along a straight line by a continuous function $f(x)$. Hooke's Law gave a nice application of such a function. Be able to work problems similar to the homework, including the "tank" problems. The second topic we covered in this section was hydrostatic pressure. Know how pressure is defined and find the force due to hydrostatic pressure exerted on a vertical plate submerged in a fluid.

Section 7.6 We defined differential equation, order, and solution and we learned to solve separable equations (for general solutions and initial-value problems). Important examples included finding orthogonal trajectories, the law of natural growth, and logistic equations.

Section 8.1 Definition of a sequence, a limit of a sequence, convergence/divergence, and increasing/decreasing (monotonic) sequences. To evaluate limits, we use the Limit Laws, the Squeeze Theorem, the Monotone Sequence Theorem, or one of several other theorems given in this section. In particular, be familiar with the values of r for which $\{r^n\}$ converges.

Section 8.2 Definition of a series, partial sum, convergence/divergence, geometric series, harmonic series, and telescoping series. You should know how to determine whether or not a geometric series converges and evaluate it when it does converge. The test for divergence was also given in this section.

Section 8.3 This section introduced the Integral Test, Comparison Test, and Limit Comparison Test for series. Make sure that you carefully check that a series satisfies the hypotheses of these tests before using them. You should be familiar with the specific examples of the harmonic series (or more generally, p-series), geometric series, and telescoping series.

Section 8.4 We defined absolutely convergent and conditionally convergent and stated the Alternating Series Test, Ratio Test, and Root Test. One important series that you should be familiar with are alternating p-series.

Section 8.5 We defined power series, power series centered at a, radius of convergence, and interval of convergence. A theorem was given which described the possible radii of convergence and we worked many examples.

Section 8.6 Beginning with the power series representation

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \qquad |x| < 1,$$

we found representations for many other related functions. In some cases, this process made use of the theorem which allows us to differentiate or integrate power series term-by-term.

Section 8.7 When a function has a power series representation centered at a, the Taylor series gave us a method of finding it. Maclaurin series are just the special case where $a = 0$. Assuming that they exist, be able to find Taylor (Maclaurin) series for functions similar to those given in class.

Section 9.1 We graphed curves defined by parametric equations. One important example was that of a cycloid, the curve traced by a fixed point on a circle as the circle rolls along the x-axis.

Section 9.2 If a curve is defined by parametric equations, you should be able to find equations of tangent lines, points at which the tangent is horizontal or vertical, the area under the curve, and the length of part of the curve. We concluded the section by working out an area of a surface of revolution problem when a curve is given parametrically.

Section 9.3 We began the section by learning how to convert between the polar and Cartesian coordinate systems. In particular, you should be aware that every point in the plane has infinitely-many polar coordinate representations, but only one Cartesian coordinate representation. Be able to graph curves given in polar coordinates and find derivatives.

Section 9.4 Be able to find areas and lengths when curves are described in polar coordinates.

Section 9.5 We defined the three conic sections: parabolas, ellipses, and hyperbolas. From these geometric definitions, we derived the standard (Cartesian) equations. In addition to these equations, you should be familiar with their polar coordinate descriptions.

Other In addition to the topics listed above, you should be able to work problems similar to the examples covered in class and the assigned homework problems listed below.

ASSIGNED HOMEWORK PROBLEMS

Section 6.1 # 3-6, 9, 13, 17, 25, 29, 30

Section 6.2 # 1, 3, 5, 13, 17, 21, 23, 29, 31, 33, 41, 45, 47, 51, 59, 61

Section 6.3 # 1, 3, 5, 7, 9, 13, 15, 19, 21, 25, 35, 39, 41

Section 6.5 # 5, 7, 11

Section 6.6 # 3, 5, 7, 9, 11, 13, 17, 19, 29, 33, 34, 41, 42, 47, 51

Section 7.2 # 1, 3, 5, 7, 9, 11, 13, 25, 26, 32, 34, 35, 41

Section 7.3 # 3, 5, 7-9, 11, 13, 15, 17, 33, 37

Section 7.4 # 3, 5, 7, 9, 13, 26

Areas of Surfaces of Revolution Handout

Section 7.5 # 6-8, 10, 15, 17, 23, 27, 30

Section 7.6 # 1, 3, 5, 7, 9, 11, 13, 15, 21-24, 35, 37, 40

Section 8.1 # 3, 5, 7, 9, 11, 13, 15, 17, 19, 27, 33, 41

Section 8.2 # 3, 5, 7, 9, 11, 13, 19, 21, 23, 24, 27, 38

Section 8.3 # 3, 4, 6, 11, 13, 19, 21

Section 8.4 # 5, 7, 18, 19, 21, 23, 25, 27, 31, 37, 40

Section 8.5 # 3, 5, 7, 9, 11, 13, 17, 19, 21

Section 8.6 # 3, 5, 9, 11, 13, 15, 17, 19, 32, 35

Section 8.7 # 3-8, 11, 15, 16, 27, 28

Section 9.1 # 1, 3, 5, 7, 9, 13, 19

Section 9.2 # 3, 5, 7, 9, 11, 13, 15, 21, 28, 29, 33, 35, 39

Section 9.3 # 1, 3, 5, 9, 13, 15, 17, 23, 25, 27, 31, 35, 39, 47, 49, 51, 53, 55, 56

Section 9.4 # 1, 3, 5-9, 11, 15, 19, 21, 29, 33, 35

Section 9.5 # 1, 3, 5, 7, 9, 11, 13, 15