

MATHEMATICS RESEARCH SEMINAR

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REGARDING THE EXISTENCE OF COINCIDENCE POINTS

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ABSTRACT: A function $f : X \rightarrow X$ has a fixed point if there exists $x \in X$ so that $f(x) = x$. In 1969, R.H. Bing published “The Elusive Fixed Point Property”, an article leading to some of the most difficult unsolved problems of point set topology at the time (and still to this day). The paper spawned numerous results regarding fixed points, their generalizations, and applications. The topic of this talk regards a related notion: that of the existence of coincidence points for commuting maps (continuous functions). Two functions $f : X \rightarrow X$ and $g : X \rightarrow X$ commute if $f(g(x)) = g(f(x))$ for each $x \in X$. The point x_0 is a coincidence point of f and g if $f(x_0) = g(x_0)$. The study of coincidence points stems from issues about whether it is necessary that two commuting maps on nice spaces have a common fixed point. As shown in 1969 by W. Boyce, there exist commuting maps of the interval to itself which do not have a common fixed point. It is true, however that commuting maps on an interval must have a coincidence point; a short proof will be given. An open question about whether the same is true for the triod remains.