26. There are a total of 9000 four-digit positive integers. The number of these that do not contain the digit 1 is \(8 \cdot 9 \cdot 9 \cdot 9 = 5832\), so there are \(9000 - 5832 = 3168\) four-digit positive integers that contain the digit 1.

32. He can either work on no Sundays or on exactly one Sunday. There are \(\binom{26}{5} = 65,780\) ways to work on no Sundays, and \(\binom{5}{1} \cdot \binom{26}{4} = 74,750\) to work on exactly one Sunday, for a total of 140,530 ways.

35. There are a total of \(2^n \cdot 2^n = 4^n\) ways to pick any two subsets of \([n]\). We count the number of ways to select two subsets \(A\) and \(B\) that are disjoint. There is a bijection between pairs of disjoint subsets of \([n]\) and \(n\)-digit strings over the 3-letter alphabet \(\{A, B, C\}\) as follows. Let the \(k\)th digit of the string be \(A\) if \(k\) is in \(A\), \(B\) if \(k\) is in \(B\), and \(C\) if \(k\) is in neither \(A\) nor \(B\). Since \(A\) and \(B\) are disjoint, \(k\) cannot be in both \(A\) and \(B\). Thus, there are \(3^n\) ways to select two disjoint subsets of \([n]\), and hence \(4^n - 3^n\) ways to select two subsets \(A\) and \(B\) of \([n]\) that are not disjoint.

39. Each point of intersection (in the interior of the polygon) lies on exactly two diagonals; those diagonals cannot share an endpoint, or else the point of intersection would not be in the interior. Thus, each point of intersection corresponds to a set of four distinct vertices of the polygon (the endpoints of the two diagonals). Conversely, given any four distinct vertices of the polygon, there is only one way to connect them in pairs so that the diagonals intersect. Therefore, there is a bijection between the points of intersection of the diagonals and four-element sets of vertices, so the diagonals have \(\binom{n}{4}\) points of intersection.

41. Brenda wins if none of the four dice shows a six; this probability is \(\left(\frac{5}{6}\right)^4 = \frac{625}{1296}\), which is about 0.482. Andy wins otherwise, with probability around 0.518, so Andy has a greater chance of winning.

Bonus Problem:
(a) If \(k \leq 5\), then the number of sequences with \(k\) non-consecutive heads in 10 tosses is given by \(\binom{10-k+1}{k}\), according to the penultimate formula on the summary sheet, or the method of Example 3.19 in the text. This gives
\[
\binom{11}{0} + \binom{10}{1} + \binom{9}{2} + \binom{8}{3} + \binom{7}{4} + \binom{6}{5} = 1 + 10 + 36 + 56 + 35 + 6 = 144
\]
sequences with no consecutive heads.

(b) With 11 tosses, there can be as many as 6 nonconsecutive heads, giving
\[
\sum_{k=0}^{6} \binom{11 - k + 1}{k} = \binom{12}{0} + \binom{11}{1} + \binom{10}{2} + \binom{9}{3} + \binom{8}{4} + \binom{7}{5} + \binom{6}{6} \\
= 1 + 11 + 45 + 84 + 70 + 21 + 1 = 233
\]
possible sequences with no consecutive heads.

(c) With \(n\) tosses, there are \(F_{n+2}\) sequences with no consecutive heads, where \(F_n\) is the \(n\)th Fibonacci number, defined by \(F_0 = 1 = F_1\) and \(F_n = F_{n-1} + F_{n-2}\) for \(n \geq 3\). Later, we will give a careful proof of the following formula:
\[
\sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n-k}{k} = F_{n+1}.
\]
Thus, with \(n\) tosses, there are \(\sum_{k=0}^{\lfloor (n+1)/2 \rfloor} \binom{n-k+1}{k} = F_{n+2}\) sequences with no consecutive heads.